DIAGNOSTIC IN-CLASS QUIZ: DUE MONDAY NOVEMBER 25: SUBSPACE, BASIS, AND DIMENSION

MATH 196, SECTION 57 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________________

PLEASE DO NOT DISCUSS *ANY* QUESTIONS.

This quiz covers material related to the `Linear dependence, bases and subspaces` notes corresponding to Sections 3.2 and 3.3 of the text.

Keep in mind the following facts. Suppose $T : \mathbb{R}^m \to \mathbb{R}^n$ is a linear transformation. Suppose A is the matrix for T, so that $T(\vec{x}) = A\vec{x}$ for all $\vec{x} \in \mathbb{R}^m$. Then, A is a $n \times m$ matrix. Further, the following are true:

- The dimension of the image of T equals the rank of A.
- The dimension of the kernel of T, called the *nullity* of A, is m minus the rank of A.

(1) *Do not discuss this!*: Suppose $T : \mathbb{R}^m \to \mathbb{R}^n$ is a linear transformation. What is the best we can say about the dimension of the image of T?
(A) It is at least 0 and at most $\min\{m, n\}$. However, we cannot be more specific based on the given information.
(B) It is at least 0 and at most $\max\{m, n\}$. However, we cannot be more specific based on the given information.
(C) It is at least $\min\{m, n\}$ and at most $\max\{m, n\}$. However, we cannot be more specific based on the given information.
(D) It is at least $\min\{m, n\}$ and at most $m + n$. However, we cannot be more specific based on the given information.
(E) It is at least $\max\{m, n\}$ and at most $m + n$. However, we cannot be more specific based on the given information.

Your answer: ______________________

(2) *Do not discuss this!*: Suppose $T_1, T_2 : \mathbb{R}^m \to \mathbb{R}^n$ are linear transformations. Suppose the images of T_1 and T_2 have dimensions d_1 and d_2 respectively. What can we say about the dimension of the image of $T_1 + T_2$? Assume that both m and n are larger than $d_1 + d_2$.
(A) It is precisely $|d_2 - d_1|$.
(B) It is precisely $\min\{d_1, d_2\}$.
(C) It is precisely $\max\{d_1, d_2\}$.
(D) It is precisely $d_1 + d_2$.
(E) Based on the information, it could be any integer r with $|d_2 - d_1| \le r \le d_1 + d_2$.

Your answer: ______________________

(3) *Do not discuss this!*: Suppose V_1 and V_2 are subspaces of $\mathbb{R}^n$. We define the sum $V_1 + V_2$ as the subset of $\mathbb{R}^n$ comprising all vectors that can be expressed as a sum of a vector in V_1 and a vector in V_2. Define $V_1 \cup V_2$ as the set-theoretic union of V_1 and V_2, i.e., the set of all vectors that are either in V_1 or in V_2. What can we say about these?
(A) $V_1 \cup V_2 = V_1 + V_2$ and it is a subspace of $\mathbb{R}^n$.
(B) $V_1 \cup V_2$ is contained in $V_1 + V_2$ and both are subspaces of $\mathbb{R}^n$.
(C) $V_1 \cup V_2$ is contained in $V_1 + V_2$, and $V_1 + V_2$ is a subspace of $\mathbb{R}^n$. $V_1 \cup V_2$ is generally not a subspace of $\mathbb{R}^n$ (though it might be in special cases).
(D) $V_1 \cup V_2$ contains $V_1 + V_2$, and both are subspaces of $\mathbb{R}^n$.
(E) $V_1 \cup V_2$ contains $V_1 + V_2$, and $V_1 \cup V_2$ is a subspace of $\mathbb{R}^n$. $V_1 + V_2$ is generally not a subspace of $\mathbb{R}^n$ (though it might be in special cases).

Your answer: ______________________