DIAGNOSTIC IN-CLASS QUIZ: ORIGINALLY DUE FRIDAY NOVEMBER 15, DELAYED TO WEDNESDAY NOVEMBER 20: LINEAR DEPENDENCE, BASES, AND SUBSPACES

MATH 196, SECTION 57 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

PLEASE DO NOT DISCUSS *ANY* QUESTIONS

The purpose of this quiz is to review some basic ideas from part of the lecture notes titled `Linear dependence, bases, and subspaces`. The corresponding sections of the book are Sections 3.2 and 3.3.

(1) *Do not discuss this!*: Suppose S is a finite nonempty set of vectors in $\mathbb{R}^n$, and T is a nonempty subset of S. What can we say about S and T?
  (A) S is linearly dependent if and only if T is linearly dependent. S is linearly independent if and only if T is linearly independent.
  (B) If S is linearly dependent, then T is linearly dependent. If S is linearly independent, then T is linearly independent. However, we cannot deduce anything about the linear dependence or independence of S from the linear dependence or independence of T.
  (C) If T is linearly dependent, then S is linearly dependent. If T is linearly independent, then S is linearly independent. However, we cannot deduce anything about the linear dependence or independence of T from the linear dependence or independence of S.
  (D) If S is linearly dependent, then T is linearly dependent. If T is linearly independent, then S is linearly independent. We cannot make either of the two other deductions.
  (E) If T is linearly dependent, then S is linearly dependent. If S is linearly independent, then T is linearly independent. We cannot make either of the other two deductions.

  Your answer: ____________________________

(2) *Do not discuss this!*: Suppose S is a finite set of vectors in $\mathbb{R}^n$. Consider the three statements: (i) S is linearly independent, (ii) S does not contain the zero vector, (iii) S does not contain any two vectors that are scalar multiples of one another. Which of the following options best describes the relationship between these statements?
  (A) (i) is equivalent to (ii), and both imply (iii), but the reverse implication does not hold.
  (B) (i) is equivalent to (iii), and both imply (ii), but the reverse implication does not hold.
  (C) (i) is equivalent to (ii) and (iii) combined.
  (D) (i) implies both (ii) and (iii), but (ii) and (iii), even if combined, do not imply (i).

  Your answer: ____________________________

(3) *Do not discuss this!*: Suppose V is a linear subspace of $\mathbb{R}^n$ for some n, and W is a linear subspace of V. Assume also that $W \neq V$, i.e., W is a *proper* subspace of V. Which of the following correctly describes the relationship between bases of V and bases of W?
  (A) Given a basis of V, we can find a subset of that basis that is a basis of W. Also, given a basis of W, we can find a set containing that basis that is a basis of V.
  (B) Given a basis of V, we can find a subset of that basis that is a basis of W. However, given a basis of W, we may not necessarily be able to find a set containing that basis that is a basis of V.
  (C) Given a basis of V, we may not necessarily be able to find a subset of that basis that is a basis of W. However, given a basis of W, we can find a set containing that basis that is a basis of V.

  Your answer: ____________________________