DIAGNOSTIC IN-CLASS QUIZ: DUE FRIDAY NOVEMBER 8: IMAGE AND KERNEL (COMPUTATIONAL)

MATH 196, SECTION 57 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

PLEASE DO NOT DISCUSS *ANY* QUESTIONS.

The questions here test for an understanding of the ideas covered in the lecture notes titled `Image and kernel of a linear transformation`. However, the format of presentation of the questions in the quiz differs somewhat from that used in typical linear algebra problems, so you need to think a bit before plugging and chugging. The corresponding section of the book is Section 3.1.

All these questions can be solved without using any part of the "toolkit" of linear algebra, but they can be understood better and more deeply using the ideas and methods of linear algebra.

(1) *Do not discuss this!*: Consider the linear transformation $\text{Avg} : \mathbb{R}^2 \to \mathbb{R}^2$ defined as:

$$\text{Avg} = \begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} (x+y)/2 \\ (x+y)/2 \end{bmatrix}$$

What can we say about the kernel and image of Avg? Note that in our descriptions of the kernel and the image below, we use x to denote the first coordinate of the vector and y to denote the second coordinate of the vector.

Note: One way you can do that is to write the matrix for Avg, but in this particular situation, it's easiest to just do things directly.

(A) The kernel is the zero subspace and the image is all of $\mathbb{R}^2$
(B) The kernel is the line $y = x$ and the image is also the line $y = x$
(C) The kernel is the line $y = x$ and the image is the line $y = -x$
(D) The kernel is the line $y = -x$ and the image is also the line $y = -x$
(E) The kernel is the line $y = -x$ and the image is the line $y = x$

Your answer: ____________________________

(2) *Do not discuss this!*: Consider the *average of other two* linear transformation $\nu : \mathbb{R}^3 \to \mathbb{R}^3$ given as follows:

$$\nu = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} (y+z)/2 \\ (z+x)/2 \\ (x+y)/2 \end{bmatrix}$$

What can we say about the kernel and image of ν?

Note that in our descriptions of the kernel and the image below, we use x to denote the first coordinate of the vector, y to denote the second coordinate of the vector, and z to denote the third coordinate of the vector.

Note: This can both be reasoned directly (without any knowledge of linear algebra) or alternatively it can be done by writing the matrix of ν and computing its rank, image, and kernel.

(A) The kernel is the zero subspace and the image is all of $\mathbb{R}^3$
(B) The kernel is the line $x = y = z$ (one-dimensional) and the image is the plane $x + y + z = 0$ (two-dimensional)
(C) The kernel is the plane $x + y + z = 0$ (two-dimensional) and the image is the line $x = y = z$ (one-dimensional)
(D) The kernel is the plane $x = y = z$ (two-dimensional) and the image is the line $x + y + z = 0$ (one-dimensional)

(E) The kernel is the line $x + y + z = 0$ (one-dimensional) and the image is the plane $x = y = z$ (two-dimensional)

Your answer: ____________________________

(3) *Do not discuss this!*: Consider the *difference of other two* linear transformation $\mu : \mathbb{R}^3 \to \mathbb{R}^3$ given by:

$$\mu = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \mapsto \begin{bmatrix} y - z \\ z - x \\ x - y \end{bmatrix}$$

What can we say about the kernel and image of μ?

Note that in our descriptions of the kernel and the image below, we use x to denote the first coordinate of the vector, y to denote the second coordinate of the vector, and z to denote the third coordinate of the vector.

Note: This can both be reasoned directly (without any knowledge of linear algebra) or alternatively it can be done by writing the matrix of μ and computing its rank, image, and kernel.

(A) The kernel is the zero subspace and the image is all of $\mathbb{R}^3$

(B) The kernel is the line $x = y = z$ (one-dimensional) and the image is the plane $x + y + z = 0$ (two-dimensional)

(C) The kernel is the plane $x + y + z = 0$ (two-dimensional) and the image is the line $x = y = z$ (one-dimensional)

(D) The kernel is the plane $x = y = z$ (two-dimensional) and the image is the line $x + y + z = 0$ (one-dimensional)

(E) The kernel is the line $x + y + z = 0$ (one-dimensional) and the image is the plane $x = y = z$ (two-dimensional)

Your answer: ____________________________