TAKE-HOME CLASS QUIZ: DUE FRIDAY NOVEMBER 1: MATRIX MULTIPLICATION AND INVERSION: ABSTRACT BEHAVIOR PREDICTION

MATH 196, SECTION 57 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________________

PLEASE FEEL FREE TO DISCUSS *ALL* QUESTIONS.

This quiz tests for *abstract behavior prediction* related to the structure of matrices defined based on the operations of matrix multiplication and inversion. It is based on part of the `Matrix multiplication and inversion` notes and is related to Sections 2.3 and 2.4. It does not, however, test all aspects of that material.

To understand this abstract behavior, we will consider *nilpotent*, *invertible*, and *idempotent* matrices.

(1) Suppose A and B are $n \times n$ matrices such that B is invertible. Suppose r is a positive integer. What can we say that $(BAB^{-1})^r$ definitely equals?
(A) A^r
(B) BA^rB^{-1}
(C) $B^rA^rB^{-r}$
(D) B^rAB^{-r}
(E) BAB^{-1-r}

Your answer: ____________________________

(2) Suppose A and B are $n \times n$ matrices (n not too small) such that $(AB)^2 = 0$. What is the smallest r for which we can conclude that $(BA)^r$ is definitely 0?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Your answer: ____________________________

(3) Suppose $n > 1$. A $n \times n$ matrix A is termed *nilpotent* if there exists a positive integer r such that A^r is the zero matrix. It turns out that if A is nilpotent, then $A^n = 0$. Which of the following describes correctly the relationship between being invertible and being nilpotent for $n \times n$ matrices?
(A) A matrix is nilpotent if and only if it is invertible.
(B) Every nilpotent matrix is invertible, but not every invertible matrix is nilpotent.
(C) Every invertible matrix is nilpotent, but not every nilpotent matrix is invertible.
(D) An invertible matrix may or may not be nilpotent, and a nilpotent matrix may or may not be invertible.
(E) A matrix cannot be both nilpotent and invertible.

Your answer: ____________________________

(4) Suppose A and B are $n \times n$ matrices. Which of the following is true? Please see Option (E) before answering.
(A) AB is nilpotent if and only if A and B are both nilpotent.
(B) AB is nilpotent if and only if at least one of A and B is nilpotent.
(C) If both A and B are nilpotent, then AB is nilpotent, but AB being nilpotent does not imply that both A and B are nilpotent.
(D) If AB is nilpotent, then both A and B are nilpotent. However, both A and B being nilpotent does not imply that AB is nilpotent.

(E) None of the above.

Your answer: ____________________________

(5) Suppose A and B are $n \times n$ matrices. Which of the following is true? Please see Option (E) before answering.

(A) AB is invertible if and only if A and B are both invertible.

(B) AB is invertible if and only if at least one of A and B is invertible.

(C) If both A and B are invertible, then AB is invertible, but AB being invertible does not imply that both A and B are invertible.

(D) If AB is invertible, then both A and B are invertible. However, both A and B being invertible does not imply that AB is invertible.

(E) None of the above.

Your answer: ____________________________

(6) Suppose A and B are $n \times n$ matrices. Which of the following is true? We call a $n \times n$ matrix *idempotent* if it equals its own square. Please see Option (E) before answering.

(A) AB is idempotent if and only if A and B are both idempotent.

(B) AB is idempotent if and only if at least one of A and B is idempotent.

(C) If both A and B are idempotent, then AB is idempotent, but AB being idempotent does not imply that both A and B are idempotent.

(D) If AB is idempotent, then both A and B are idempotent. However, both A and B being idempotent does not imply that AB is idempotent.

(E) None of the above.

Your answer: ____________________________