TAKE-HOME CLASS QUIZ: DUE FRIDAY OCTOBER 11: LINEAR SYSTEMS

MATH 196, SECTION 57 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

YOU MAY DISCUSS ALL QUESTIONS, BUT PLEASE ENTER FINAL ANSWER OPTIONS THAT YOU ARE PERSONALLY MOST CONVINCED OF. BEWARE OF GROUP-THINK!

The quiz questions here, although not hard *per se*, are conceptually demanding: answering them requires a clear understanding of multiple concepts and an ability to execute them conjunctively. Even if you feel that you've understood the material as presented in class, you will need to think through each question carefully. Some of the questions are related to similar homework problems (Homeworks 1 and 2), and they test a conceptual understanding of the solutions to these problems. You might want to view them in conjunction with the homework problems. Other questions sow the seeds of ideas we will explore later. The quiz should seem relatively easier when you review it later, assuming that you work hard on attempting the questions right now and read the solutions once they're put up.

(1) (*) Rashid and Riena are trying to study a function f of two variables x and y. Rashid is convinced that the function is linear (i.e., it is of the form $f(x, y) := ax + by + c$) but has no idea what a, b, and c are. Riena thinks a linear model is completely out-of-place. Rashid is eager to find a, b, and c, whereas Riena is eager to disprove Rashid's linear model. Unfortunately, all they have is a black box that will output the value of the function for a given input pair (x, y), and that black box can only be called three times. What should Rashid and Riena try for?
(A) Rashid and Riena would both like to provide three input pairs that are non-collinear as points in the xy-plane
(B) Rashid would like to provide three input pairs that are non-collinear, while Riena would like to provide three input pairs that are collinear as points in the xy-plane.
(C) Rashid and Riena would both like to provide three input pairs that are collinear as points in the xy-plane.
(D) Rashid would like to provide three input pairs that are collinear, while Riena would like to provide three input pairs that are non-collinear as points in the xy-plane.
(E) Both Rashid and Riena are indifferent regarding how the three input pairs are picked.

Your answer: ______________________

(2) (*) Let m and n be natural numbers with $m \geq 3$. We are given a bunch of numbers $x_1 < x_2 < \dots < x_m$ and another bunch of numbers $y_1, y_2, \dots, y_m$. We want to find a continuous function f on $[x_1, x_m]$, such that $f(x_i) = y_i$ for all $1 \leq i \leq m$, and such that the restriction of f to any interval of the form $[x_i, x_{i+1}]$ (for $1 \leq i \leq m - 1$) is a polynomial of degree $\leq n$. What is the smallest value of n for which we are guaranteed to be able to find such a function f?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Your answer: ______________________

(3) (*) Let m and n be natural numbers with $m \geq 3$. We are given a bunch of numbers $x_1 < x_2 < \dots < x_m$ and another bunch of numbers $y_1, y_2, \dots, y_m$. We want to find a continuous function f on $[x_1, x_m]$, such that $f(x_i) = y_i$ for all $1 \leq i \leq m$, and such that the restriction of f to any interval of

the form $[x_i, x_{i+1}]$ (for $1 \le i \le m-1$) is a polynomial of degree $\le n$. In addition, we want to make sure that f is differentiable on the open interval (x_1, x_m). What is the smallest value of n for which we are guaranteed to be able to find such a function f?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Your answer: ____________________________

(4) (*) Let m and n be natural numbers with $m \ge 3$. We are given a bunch of numbers $x_1 < x_2 < \dots < x_m$ and another bunch of numbers $y_1, y_2, \dots, y_m$. We want to find a continuous function f on $[x_1, x_m]$, such that $f(x_i) = y_i$ for all $1 \le i \le m$, and such that the restriction of f to any interval of the form $[x_i, x_{i+1}]$ (for $1 \le i \le m-1$) is a polynomial of degree $\le n$. In addition, we want to make sure that f is differentiable on the open interval (x_1, x_m). In addition, we are told the value of the right hand derivative of f at x_1 and the left hand derivative of f at x_m. What is the smallest value of n for which we are guaranteed to be able to find such a function f?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Your answer: ____________________________

(5) (*) Let k, m, and n be natural numbers with $m \ge 3$. We are given a bunch of numbers $x_1 < x_2 < \dots < x_m$ and another bunch of numbers $y_1, y_2, \dots, y_m$. We want to find a continuous function f on $[x_1, x_m]$, such that $f(x_i) = y_i$ for all $1 \le i \le m$, and such that the restriction of f to any interval of the form $[x_i, x_{i+1}]$ (for $1 \le i \le m-1$) is a polynomial of degree $\le n$. In addition, we want to make sure that f is at least k times differentiable on the open interval (x_1, x_m). What is the smallest value of n for which we are guaranteed to be able to find such a function f?
(A) $k-2$
(B) $k-1$
(C) k
(D) $k+1$
(E) $k+2$

Your answer: ____________________________

The next few questions are framed deterministically, though similar real-world applications would be probabilistic, with some square roots floating around. Unfortunately, we do not have the tools yet to deal with the probabilistic versions of the questions.

(6) (*) A function f of one variable is known to be linear. We know that $f(1) = 1.5 \pm 0.5$ and $f(2) = 2.5 \pm 0.5$. Assume these are the full ranges, without any probability distribution known. Assuming nothing is known about how the measurement errors for f at different points are related, what can we say about $f(3)$?
(A) $f(3) = 3.5$ (exactly)
(B) $f(3) = 3.5 \pm 0.5$
(C) $f(3) = 3.5 \pm 1$
(D) $f(3) = 3.5 \pm 1.5$
(E) $f(3) = 3.5 \pm 2.5$

Your answer: ____________________________

(7) (*) A function f of one variable is known to be linear. We know that $f(1) = 1.5 \pm 0.5$ and $f(2) = 2.5 \pm 0.5$. Assume these are the full ranges, without any probability distribution known. Assume also that the measurement error for f at all points is the same in magnitude and sign. What can we say about $f(3)$?

(A) $f(3) = 3.5$ (exactly)
(B) $f(3) = 3.5 \pm 0.5$
(C) $f(3) = 3.5 \pm 1$
(D) $f(3) = 3.5 \pm 1.5$
(E) $f(3) = 3.5 \pm 2.5$

Your answer: ____________________________

(8) (*) Suppose f is a linear function on a bounded interval $[a, b]$ but our measurement of outputs for given inputs has some measurement error (with the range of measurement error the same regardless of the input, and no known correlation between the magnitude of measurement error at different points). Assume we can get the outputs for any two specified inputs we desire, and we will then fit a line through the (input,output) pairs to get the graph of f. How should we choose our inputs?

(A) Choose the inputs as far as possible from each other, i.e., choose them as a and b.
(B) Choose the inputs to be as close to each other as possible, i.e., choose them to be nearby points but not equal to each other.
(C) It does not matter. Any choice of two distinct inputs is good enough.

Your answer: ____________________________

(9) (*) f is a function of one variable defined on an interval $[a, b]$. You are trying to find an explicit function that fits f well. You initially try a straight line fit that works at the points a and b. It turns out that this fit systematically overestimates f for points in between (i.e., the actual function f is below the linear function) with the maximum magnitude of discrepancy occurring at the midpoint $(a + b)/2$. Based on this information, what kind of fit should you try to look for?

(A) Try to fit f using a logarithmic function
(B) Try to fit f using an exponential function
(C) Try to fit f using a quadratic function
(D) Try to fit f using a polynomial of degree at most 3
(E) Try to fit f using the reciprocal of a linear function

Your answer: ____________________________

(10) (*) Recall the Leontief input-output model. Recall that the GDP is defined as the total money value of all the *final* goods and services produced in the economy, which in this case means only those that go into meeting consumer demand, not interindustry demand (note that we are assuming away the existence of investment and government spending, which complicate the GDP calculation). Assuming that the unit prices of the goods are constant (a very unrealistic assumption given that price itself responds to supply and demand, but fortunately it does not affect the conclusion we draw here) what might be a way of increasing GDP while keeping the magnitude of output of each industry the same?

(A) Increase interindustry dependence, i.e., increase the amount needed from each industry that is necessary to produce a given amount in another industry.
(B) Reduce interindustry dependence, i.e., reduce the amount needed from each industry that is necessary to produce a given amount in another industry.
(C) Changes in interindustry dependence have no effect.

Your answer: ____________________________