TAKE-HOME CLASS QUIZ: DUE FRIDAY OCTOBER 4: LINEAR FUNCTIONS AND EQUATION-SOLVING (PART 1)

MATH 196, SECTION 57 (VIPUL NAIK)

Your name (print clearly in capital letters): ____________________________________

PLEASE DO *NOT* DISCUSS ANY QUESTIONS EXCEPT THE STARRED OR DOUBLE-STARRED QUESTIONS. YOU CAN DISCUSS THE STARRED AND DOUBLE-STARRED QUESTIONS.

This quiz covers some basics involving linear functions and equation-solving (notes at `Linear functions: a primer` and `Equation-solving with a special focus on the linear case`). The quiz tests for the following:

- What it means to be (affine) linear, and in particular, the significance of the intercept as an additional parameter to track.
- The distinction between behavior relative to the variables (the inputs) and behavior relative to the parameters.
- Using the linear paradigm to study functional forms that are not themselves linear.
- A small taste of dealing with measurement uncertainty to obtain upper and lower bounds (not covered in the notes, so this is where your famed ability to think out of the box should manifest).
- Solving "triangular" systems of equations.

(1) (*) A function f of 3 variables x, y, z defined everywhere is (affine) linear in the variables. (The "affine" is to indicate that the intercept may be nonzero). Based on the above information and some input-output pairs for f, we would like to determine f uniquely. What is the minimum number of input-output pairs that we would need in order to achieve this?
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Your answer: ____________________________

(2) *Do not discuss this!*: Which of the following gives an example of a function F of three variables x, y, z whose third-order mixed partial derivative F_{xyz} is zero everywhere, but for which none of the second-order mixed partial derivatives F_{xy}, F_{xz}, F_{yz} is zero everywhere?
(A) $\sin(xy) - z^2$
(B) $\cos(x^2 + y^2) - \sin(y^2 + z^2)$
(C) $e^{xy} + (y - z)^2 + 3xz$
(D) $x^2 + y^2 + z^2$
(E) xyz

Your answer: ____________________________

(3) *Do not discuss this!*: Consider a function of the form $F(x, y) := Ca^x b^y$ where C, a, b are all positive reals that serve as parameters and x, y are restricted to the positive reals. We wish to study F using the paradigm of linear functions. What is the best way of doing this?
(A) Express $\ln(F(x, y))$ in terms of $\ln x$ and $\ln y$
(B) Express $\ln(F(x, y))$ in terms of x and y
(C) Express $F(x, y)$ in terms of $\ln x$ and $\ln y$
(D) Express $\ln(F(x, y))$ in terms of a^x and b^y
(E) Express $F(x, y)$ in terms of a^x and b^y

Your answer: ___________________________

(4) *Do not discuss this!*: Consider a function of the form $F(x, y) := Cx^a y^b$ where C, a, b are all positive reals that serve as parameters and x, y are restricted to the positive reals. We wish to study F using the paradigm of linear functions. What is the best way of doing this?
(A) Express $\ln(F(x, y))$ in terms of $\ln x$ and $\ln y$
(B) Express $\ln(F(x, y))$ in terms of x and y
(C) Express $F(x, y)$ in terms of $\ln x$ and $\ln y$
(D) Express $\ln(F(x, y))$ in terms of x^a and y^b
(E) Express $F(x, y)$ in terms of x^a and y^b

Your answer: ___________________________

(5) (**) *This is a hard question!* The population in the island of Andrognesia as a function of time is believed to be an exponential function. On January 1, 1984, the population was measured to be $3 * 10^5$ with a measurement error of up to 10^5 on either side, i.e., the population was measured to be between $2 * 10^5$ and $4 * 10^5$. On January 1, 1998, the population was measured to be $1.2 * 10^6$ with a measurememt error of up to $4 * 10^5$ on either side, i.e., the population was measured to be between $8 * 10^5$ and $1.6 * 10^6$. If the population is an exponential function of time (i.e., the increment in population per year is a fixed proportion of the population that year), what is the **range of possible values** of the population measured on January 1, 2012? *Hint: Think of the umbral versus penumbral region for an eclipse*
(A) Between $3.2 * 10^6$ and $6.4 * 10^6$
(B) Between $3.2 * 10^6$ and $1.28 * 10^7$
(C) Between $1.6 * 10^6$ and $3.2 * 10^6$
(D) Between $1.6 * 10^6$ and $6.4 * 10^6$
(E) Between $1.6 * 10^6$ and $1.28 * 10^7$

Your answer: ___________________________

(6) *Do not discuss this!*: Suppose, according to our model, a particular function $f(x, y)$ is of the form $f(x, y) = a_1 + a_2 x + a_3 y + a_4 x^2 y^2$ where a_1, a_2, a_3, a_4 are parameters. Our goal is to determine the values of the parameters a_1, a_2, a_3, a_4. We do this by collecting a number of (input,output) pairs for the function f and then setting up equations in terms of the parameters using the (input,output) pairs. What can we say about the nature of f and the nature of the system of equations that we will need to solve? *Note that "nonlinear" as used here simply means that the expression is not guaranteed*

to be linear, though it may turn out to be linear in some cases. Similarly, "non-polynomial" means not guaranteed to be polynomial, though it may turn out to be polynomial in some cases.

(A) f is a linear function of x and y, hence we need to solve a linear system of equations to determine the parameters a_1, a_2, a_3, a_4.

(B) f is a nonlinear polynomial function of x and y, hence we need to solve a nonlinear polynomial system of equations to determine the parameters a_1, a_2, a_3, a_4.

(C) f is a linear function of x and y. However, we need to solve a nonlinear polynomial system of equations to determine the parameters a_1, a_2, a_3, a_4.

(D) f is a nonlinear polynomial function of x and y. However, we need to solve a linear system of equations to determine the parameters a_1, a_2, a_3, a_4.

(E) f is a nonlinear polynomial function of x and y. However, we need to solve a non-polynomial system of equations to determine the parameters a_1, a_2, a_3, a_4.

Your answer: ____________________________

(7) *Do not discuss this!*: Consider the system of equations:

$$\begin{aligned} x^2 - x &= 2 \\ y^2 + xy &= x + 13 \end{aligned}$$

What is the number of solutions to this system for real x and y?

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8

Your answer: ____________________________

(8) *Do not discuss this!*: Consider the system of equations:

$$\begin{aligned} x^2 - x &= 2 \\ y^2 + xy &= x + 13 \\ z^2 &= xy \end{aligned}$$

What is the number of solutions to this system for real x, y, and z?

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8

Your answer: ____________________________

(9) *Do not discuss this!*: Consider the system of equations:

$$\begin{aligned} x^2 - x &= 2 \\ y^2 + xy &= x + 13 \\ z^2 &= x^2 - y^2 \end{aligned}$$

What is the number of solutions to this system for real x, y, and z?

(A) 0
(B) 2
(C) 4
(D) 6
(E) 8

Your answer: ____________________________