TAKE-HOME CLASS QUIZ: DUE MONDAY MARCH 11: MAX-MIN VALUES: TWO-VARIABLE VERSION

MATH 195, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________________

YOU ARE FREE TO DISCUSS ALL QUESTIONS, BUT PLEASE ONLY ENTER FINAL ANSWER OPTIONS THAT YOU PERSONALLY ENDORSE. PLEASE DO *NOT* ENGAGE IN GROUPTHINK.

(1) Suppose $F(x, y) := f(x) + g(y)$, i.e., F is additively separable. Suppose f and g are differentiable functions of one variable, defined for all real numbers. What can we say about the critical points of F in its domain $\mathbb{R}^2$?
(A) F has a critical point at (x_0, y_0) iff x_0 is a critical point for f *or* y_0 is a critical point for g.
(B) F has a critical point at (x_0, y_0) iff x_0 is a critical point for f *and* y_0 is a critical point for g.
(C) F has a critical point at (x_0, y_0) iff $x_0 + y_0$ is a critical point for $f + g$, i.e., the function $x \mapsto f(x) + g(x)$.
(D) F has a critical point at (x_0, y_0) iff $x_0 y_0$ is a critical point for fg, i.e., the function $x \mapsto f(x)g(x)$.
(E) None of the above.

Your answer: ___________________________

(2) Suppose $F(x, y) := f(x)g(y)$ is a multiplicatively separable function. Suppose f and g are both differentiable functions of one variable defined for all real inputs. Consider a point (x_0, y_0) in the domain of F, which is $\mathbb{R}^2$. Which of the following is true?
(A) F has a critical point at (x_0, y_0) if and only if x_0 is a critical point for f and y_0 is a critical point for g.
(B) If x_0 is a critical point for f and y_0 is a critical point for g, then (x_0, y_0) is a critical point for F. However, the converse is not necessarily true, i.e., (x_0, y_0) may be a critical point for F even without x_0 being a critical point for f and y_0 being a critical point for g.
(C) If (x_0, y_0) is a critical point for F, then x_0 must be a critical point for f and y_0 must be a critical point for g. However, the converse is not necessarily true.
(D) (x_0, y_0) is a critical point for F if and only if *at least* one of these is true: x_0 is a critical point for f and y_0 is a critical point for g.
(E) None of the above.

Your answer: ___________________________

(3) Consider a homogeneous polynomial $ax^2 + bxy + cy^2$ of degree two in two variables x and y. Assume that at least one of the numbers a, b, and c is nonzero. What can we say about the local extreme values of this polynomial on $\mathbb{R}^2$?
(A) If $b^2 - 4ac < 0$, then the function has no local extreme values and its value is unbounded from both above and below. If $b^2 - 4ac = 0$, the function has local extreme value 0 and this is attained on a line through the origin. If $b^2 - 4ac > 0$, the function has local extreme value 0 and this is attained only at the origin.
(B) If $b^2 - 4ac < 0$, then the function has no local extreme values and its value is unbounded from both above and below. If $b^2 - 4ac > 0$, the function has local extreme value 0 and this is attained on a line through the origin. If $b^2 - 4ac = 0$, the function has local extreme value 0 and this is attained only at the origin.

(C) If $b^2 - 4ac > 0$, then the function has no local extreme values and its value is unbounded from both above and below. If $b^2 - 4ac = 0$, the function has local extreme value 0 and this is attained on a line through the origin. If $b^2 - 4ac < 0$, the function has local extreme value 0 and this is attained only at the origin.
(D) If $b^2 - 4ac > 0$, then the function has no local extreme values and its value is unbounded from both above and below. If $b^2 - 4ac < 0$, the function has local extreme value 0 and this is attained on a line through the origin. If $b^2 - 4ac = 0$, the function has local extreme value 0 and this is attained only at the origin.
(E) If $b^2 - 4ac = 0$, then the function has no local extreme values and its value is unbounded from both above and below. If $b^2 - 4ac < 0$, the function has local extreme value 0 and this is attained on a line through the origin. If $b^2 - 4ac > 0$, the function has local extreme value 0 and this is attained only at the origin.

Your answer: ____________________________

A subset of $\mathbb{R}^n$ is termed *convex* if the line segment joining any two points in the subset is completely within the subset. A function f of two variables defined on a closed convex domain is termed *quasiconvex* if given any two points P and Q in the domain, the maximum of f restricted to the line segment joining P and Q is attained at one (possibly both) of the endpoints P or Q.

There are many examples of quasiconvex functions, including linear functions (which are quasiconvex but not strictly quasiconvex) and all convex functions.

(4) What can we say about the maximum of a continuous quasiconvex function defined on the circular disk $x^2 + y^2 \le 1$?
(A) It must be attained at the center of the disk, i.e., the origin $(0, 0)$.
(B) It must be attained somewhere in the interior of the disk, but we cannot be more specific with the given information.
(C) It must be attained somewhere on the boundary circle $x^2 + y^2 = 1$. However, we cannot be more specific than that with the given information.
(D) It must be attained at one of the four points $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$.
(E) It could be attained at any point. We cannot be specific at all.

Your answer: ____________________________

(5) What can we say about the maximum of a continuous quasiconvex function defined on the square region $|x| + |y| \le 1$? This is the region bounded by the square with vertices $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$.
(A) It must be attained at the center of the square, i.e., the origin $(0, 0)$.
(B) It must be attained somewhere in the interior of the square, but we cannot be more specific with the given information.
(C) It must be attained somewhere on the boundary square $|x| + |y| \le 1$. However, we cannot be more specific than that with the given information.
(D) It must be attained at one of the four points $(1, 0)$, $(0, 1)$, $(-1, 0)$, and $(0, -1)$.
(E) It could be attained at any point. We cannot be specific at all.

Your answer: ____________________________

(6) Suppose $F(x, y) := f(x) + g(y)$, i.e., F is additively separable. Suppose f and g are continuous functions of one variable, defined for all real numbers. Which of the following statements about local extrema of F is **false**?
(A) If f has a local minimum at x_0 and g has a local minimum at y_0, then F has a local minimum at (x_0, y_0).
(B) If f has a local minimum at x_0 and g has a local maximum at y_0, then F has a saddle point at (x_0, y_0).

(C) If f has a local maximum at x_0 and g has a local minimum at y_0, then F has a saddle point at (x_0, y_0).
(D) If f has a local maximum at x_0 and g has a local maximum at y_0, then F has a local maximum at (x_0, y_0).
(E) None of the above, i.e., they are all true.

Your answer: ____________________________

(7) Suppose $F(x, y) := f(x)g(y)$ is a multiplicatively separable function. Suppose f and g are both continuous functions of one variable defined for all real inputs. Consider a point (x_0, y_0) in the domain of F, which is $\mathbb{R}^2$. Which of the following statements about local extrema is **true**?
(A) If f has a local minimum at x_0 and g has a local minimum at y_0, then F has a local minimum at (x_0, y_0).
(B) If f has a local minimum at x_0 and g has a local maximum at y_0, then F has a saddle point at (x_0, y_0).
(C) If f has a local maximum at x_0 and g has a local minimum at y_0, then F has a saddle point at (x_0, y_0).
(D) If f has a local maximum at x_0 and g has a local maximum at y_0, then F has a local maximum at (x_0, y_0).
(E) None of the above, i.e., they are all false.

Your answer: ____________________________