TAKE-HOME CLASS QUIZ: DUE FRIDAY MARCH 1: MAX/MIN VALUES: ONE-VARIABLE RECALL

MATH 195, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________________

YOU ARE FREE TO DISCUSS ALL QUESTIONS, BUT PLEASE ONLY ENTER FINAL ANSWER CHOICES THAT YOU PERSONALLY ENDORSE.

(1) Suppose f is a function defined on a closed interval $[a, c]$. Suppose that the left-hand derivative of f at c exists and equals ℓ. Which of the following implications is **true in general**?
(A) If $f(x) < f(c)$ for all $a \le x < c$, then $\ell < 0$.
(B) If $f(x) \le f(c)$ for all $a \le x < c$, then $\ell \le 0$.
(C) If $f(x) < f(c)$ for all $a \le x < c$, then $\ell > 0$.
(D) If $f(x) \le f(c)$ for all $a \le x < c$, then $\ell \ge 0$.
(E) None of the above is true in general.

Your answer: ____________________________

(2) Suppose f is a continuous function defined on an open interval (a, b) and c is a point in (a, b). Which of the following implications is **true**?
(A) If c is a point of local minimum for f, then there is a value $\delta > 0$ and an open interval $(c - \delta, c + \delta) \subseteq (a, b)$ such that f is non-increasing on $(c - \delta, c)$ and non-decreasing on $(c, c + \delta)$.
(B) If there is a value $\delta > 0$ and an open interval $(c - \delta, c + \delta) \subseteq (a, b)$ such that f is non-increasing on $(c - \delta, c)$ and non-decreasing on $(c, c + \delta)$, then c is a point of local minimum for f.
(C) If c is a point of local minimum for f, then there is a value $\delta > 0$ and an open interval $(c - \delta, c + \delta) \subseteq (a, b)$ such that f is non-decreasing on $(c - \delta, c)$ and non-increasing on $(c, c + \delta)$.
(D) If there is a value $\delta > 0$ and an open interval $(c - \delta, c + \delta) \subseteq (a, b)$ such that f is non-decreasing on $(c - \delta, c)$ and non-increasing on $(c, c + \delta)$, then c is a point of local minimum for f.
(E) All of the above are true.

Your answer: ____________________________

(3) Consider all the rectangles with perimeter equal to a fixed length $p > 0$. Which of the following **is true** for the unique rectangle which is a square, compared to the other rectangles?
(A) It has the largest area and the largest length of diagonal.
(B) It has the largest area and the smallest length of diagonal.
(C) It has the smallest area and the largest length of diagonal.
(D) It has the smallest area and the smallest length of diagonal.
(E) None of the above.

Your answer: ____________________________

PLEASE TURN OVER FOR REMAINING QUESTIONS

(4) Suppose the total perimeter of a square and an equilateral triangle is L. (We can choose to allocate all of L to the square, in which case the equilateral triangle has side zero, and we can choose to allocate all of L to the equilateral triangle, in which case the square has side zero). Which of the following statements **is true** for the sum of the areas of the square and the equilateral triangle? (The area of an equilateral triangle is $\sqrt{3}/4$ times the square of the length of its side).
(A) The sum is minimum when all of L is allocated to the square.
(B) The sum is maximum when all of L is allocated to the square.
(C) The sum is minimum when all of L is allocated to the equilateral triangle.
(D) The sum is maximum when all of L is allocated to the equilateral triangle.
(E) None of the above.

Your answer: ____________________________

(5) Suppose x and y are positive numbers such as $x + y = 12$. For **what values** of x and y is x^2y maximum?
(A) $x = 3$, $y = 9$
(B) $x = 4$, $y = 8$
(C) $x = 6$, $y = 6$
(D) $x = 8$, $y = 4$
(E) $x = 9$, $y = 3$

Your answer: ____________________________

(6) Consider the function $p(x) := x^2 + bx + c$, with x restricted to integer inputs. Suppose b and c are integers. The minimum value of p is attained either at a single integer or at two consecutive integers. Which of the following is a **sufficient condition** for the minimum to occur at two consecutive integers?
(A) b is odd
(B) b is even
(C) c is odd
(D) c is even
(E) None of these conditions is sufficient.

Your answer: ____________________________