TAKE-HOME CLASS QUIZ: DUE FRIDAY FEBRUARY 22: MULTI-VARIABLE INTEGRATION

MATH 195, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ____________________________________

YOU ARE ALLOWED TO DISCUSS *ONLY* THE STAR-MARKED QUESTIONS!

The following setup is for the first five questions only.

Suppose F is a function of two real variables, say x and t, so $F(x,t)$ is a real number for x and t restricted to suitable open intervals in the real number. Suppose, further, that F is jointly continuous (whatever that means) in x and t.

Define $f(t) := \int_0^\infty F(x,t)\,dx$. Here, while doing the integration, t is treated as a constant. x, the variable of integration, is being integrated on $[0,\infty)$.

Suppose further that f is defined and continuous for t in $(0,\infty)$.

In the next few questions, you are asked to compute the function f explicitly given the function F, for $t \in (0,\infty)$.

(1) *Do not discuss!* $F(x,t) := e^{-tx}$. Find f. *Last time:* 15/19 *correct*
(A) $f(t) = e^{-t}/t$
(B) $f(t) = e^{t}/t$
(C) $f(t) = 1/t$
(D) $f(t) = -1/t$
(E) $f(t) = -t$

Your answer: ____________________________

(2) *Do not discuss!* $F(x,t) := 1/(t^2+x^2)$. Find f. *Last time:* 13/19 *correct*
(A) $f(t) = \pi/(2t)$
(B) $f(t) = \pi/t$
(C) $f(t) = 2\pi/t$
(D) $f(t) = \pi t$
(E) $f(t) = 2\pi t$

Your answer: ____________________________

(3) *Do not discuss!* $F(x,t) := 1/(t^2+x^2)^2$. Find f. *Last time:* 13/19 *correct*
(A) $f(t) = \pi/t^3$
(B) $f(t) = \pi/(2t^3)$
(C) $f(t) = \pi/(4t^3)$
(D) $f(t) = \pi/(8t^3)$
(E) $f(t) = 3\pi/(8t^3)$

Your answer: ____________________________

(4) (*) *You can discuss this!* $F(x,t) = \exp(-(tx)^2)$. Use that $\int_0^\infty \exp(-x^2)\,dx = \sqrt{\pi}/2$. Find f. *Last time:* 7/19 *correct*
(A) $f(t) = t^2\sqrt{\pi}/2$
(B) $f(t) = t\sqrt{\pi}/2$
(C) $f(t) = \sqrt{\pi}/2$
(D) $f(t) = \sqrt{\pi}/(2t)$
(E) $f(t) = \sqrt{\pi}/(2t^2)$

Your answer: ____________________________

(5) (**) *You can discuss this!* (could confuse you if you don't understand it): In the same general setup as above (but with none of these specific Fs), which of the following is a *sufficient* condition for f to be an increasing function of t? *Last time:* 3/19 *correct*
(A) $t \mapsto F(x_0, t)$ is an increasing function of t for every choice of $x_0 \geq 0$.
(B) $x \mapsto F(x, t_0)$ is an increasing function of x for every choice of $t_0 \in (0, \infty)$.
(C) $t \mapsto F(x_0, t)$ is a decreasing function of t for every choice of $x_0 \geq 0$.
(D) $x \mapsto F(x, t_0)$ is a decreasing function of x for every choice of $t_0 \in (0, \infty)$.
(E) None of the above.

Your answer: ____________________________

(end of the setup)

(6) *Do not discuss!* Suppose f is a homogeneous polynomial of degree $d > 0$. Define g as the following function on positive reals: $g(a)$ is the double integral of f on the square $[0, a] \times [0, a]$. Assuming that $g(a)$ is not identically the zero function, which of these best describes the nature of $g(a)$? *Last time:* 12/19 *correct.*
(A) A constant times a^d
(B) A constant times a^{d+1}
(C) A constant times a^{d+2}
(D) A constant times a^{2d+1}
(E) A constant times a^{2d+2}

Your answer: ____________________________

(7) (*) *You can discuss this!* Suppose $g(x, y)$ and $G(x, y)$ are continuous functions of two variables and $G_{xy} = g$. How can the double integral $\int_s^t \int_u^v g(x, y)\, dy\, dx$ be described in terms of the values of G? *Last time:* 8/19 *correct*
(A) $G(v, t) + G(u, s) - G(u, t) - G(v, s)$
(B) $G(v, t) - G(v, s) + G(u, t) - G(u, s)$
(C) $G(t, v) + G(s, u) - G(t, u) - G(s, v)$
(D) $G(t, v) - G(s, v) + G(t, u) - G(s, u)$
(E) $G(t, v) + G(v, t) - G(s, u) - G(u, s)$

Your answer: ____________________________

(8) (**) *You can discuss this!* Suppose f is an elementarily integrable function, but $f(x^k)$ is not elementarily integrable for any integer $k > 1$ (examples are sin, exp, cos). For which of the following types of regions D are we *guaranteed to be able* to compute, in elementary function terms, the double integral $\int_D \int f(x^2)\, dA$ over the region (note that f is just a function of x, but we treat it as a function of two variables)? Please see Option (E) before answering and select that if applicable. *Last time:* 1/19 *correct.*
(A) A rectangle with vertices $(0, 0)$, $(0, b)$, $(a, 0)$, and (a, b), with $a, b > 0$.
(B) A triangle with vertices $(0, 0)$, $(0, b)$, $(a, 0)$, with $a, b > 0$.
(C) A triangle with vertices $(0, 0)$, $(0, b)$, (a, b), with $a, b > 0$.
(D) A triangle with vertices $(0, 0)$, $(a, 0)$, (a, b), with $a, b > 0$.
(E) All of the above

Your answer: ____________________________