

**TAKE-HOME CLASS QUIZ: DUE WEDNESDAY FEBRUARY 6: MULTIVARIABLE
FUNCTION BASICS CONTINUED**

MATH 195, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): _____

PLEASE FEEL FREE TO DISCUSS ALL QUESTIONS, BUT PLEASE ONLY ENTER FINAL ANSWER OPTIONS THAT YOU PERSONALLY CONSIDER MOST LIKELY TO BE CORRECT. DO NOT ENGAGE IN GROUPTHINK.

- (1) Suppose F is an additively separable function of two variables x and y that is defined everywhere, i.e., there exist functions f and g of one variable, both defined on all of \mathbb{R} , such that $F(x, y) = f(x) + g(y)$ for all $x, y \in \mathbb{R}$.

We call two curves *parallel* if there is a vector by which we can translate all the points in one curve to get precisely the other curve.

Consider the following three statements:

(i) All curves obtained as the intersections of the graph of F with planes parallel to the xy -plane are parallel to each other.

(ii) All curves obtained as the intersections of the graph of F with planes parallel to the xz -plane are parallel to each other.

(iii) All curves obtained as the intersections of the graph of F with planes parallel to the yz -plane are parallel to each other.

Which of the statements (i)-(iii) is/are necessarily true?

- (A) All of (i), (ii), and (iii) are true.
(B) Both (i) and (ii) are true but (iii) need not be true.
(C) Both (ii) and (iii) are true but (i) need not be true.
(D) Both (i) and (iii) are true but (ii) need not be true.
(E) (i) is true but (ii) and (iii) need not be true.

Your answer: _____

- (2) Suppose f is a continuous function of two variables x and y , defined on the entire xy -plane. Suppose further that f is increasing in x for each fixed value of y , and that f is increasing in y for every fixed value of x . Which of the following is the most plausible description of the level curves of f in the xy -plane? *Note: You might wish to take an extremely simple example, e.g., an additively separable function where each of the pieces is the simplest possible increasing function you can think of.*

- (A) They are all upward-sloping, i.e., they are of the form $y = g(x)$ with g an increasing function.
(B) They are all downward-sloping, i.e., they are of the form $y = g(x)$ with g a decreasing function.
(C) They look like closed loops (e.g., circles).
(D) They look like graphs of functions with a unique local and absolute minimum (such as the parabola $y = x^2$, though the actual picture may be different).
(E) They look like graphs of functions with a unique local and absolute maximum (such as the parabola $y = -x^2$, though the actual function may be different).

Your answer: _____

- (3) What do the level curves of the function $f(x, y) := \sin(x + y)$ look like for output value in $[-1, 1]$? Note that all these level curves are being considered as curves in the xy -plane. *Note: This builds upon the idea of Question 3 of the previous quiz.*

- (A) Each level curve is a single line.
(B) Each level curve is a union of two intersecting lines.

- (C) Each level curve is a union of two distinct parallel lines.
- (D) Each level curve is a union of infinitely many concurrent lines (i.e., infinitely many lines, all passing through the same point).
- (E) Each level curve is a union of infinitely many distinct parallel lines (i.e., infinitely many lines, all parallel to each other).

Your answer: _____

- (4) Suppose f and g are both continuous functions of two variables x and y , both defined on all of \mathbb{R}^2 , and such that $f(x, y) + g(x, y)$ is a constant C . What is the relation between the level curves of f and the level curves of g , all drawn in the xy -plane?
- (A) Every level curve of f is a level curve of g and vice versa, with the same level value for both functions.
 - (B) Every level curve of f is a level curve of g and vice versa, but the value for which it is a level curve may be different for the two functions.
 - (C) The level curves of f need not be precisely the same as the level curves of g , but we can go from one set of level curves to the other via a parallel translation.
 - (D) Each level curve of f can be obtained by reflecting a suitable level curve of g about a suitable line in the xy -plane.
 - (E) Each level curve of f can be obtained by reflecting a suitable level curves of g about a suitable line in the xy -plane and then performing a suitable translation.

Your answer: _____