CLASS QUIZ: WEDNESDAY FEBRUARY 6: MULTIVARIABLE LIMIT COMPUTATIONS

MATH 195, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ________________________________

PLEASE FEEL FREE TO DISCUSS ALL QUESTIONS, BUT PLEASE ONLY ENTER FINAL ANSWER OPTIONS THAT YOU PERSONALLY CONSIDER MOST LIKELY TO BE CORRECT. DO NOT ENGAGE IN GROUPTHINK.

(1) (**) Consider the function $f(x, y) := x \sin(1/(x^2 + y^2))$, defined on all points other than the point $(0, 0)$. What is the limit of the function at $(0, 0)$? *Last time:* 8/22 *correct*
(A) 0
(B) $1/\sqrt{2}$
(C) 1
(D) The limit is undefined, because the expression becomes unbounded around 0.
(E) The limit is undefined, because the expression is oscillatory around 0.

Your answer: ________________________

(2) The typical $\varepsilon - \delta$ definition of limit in two dimensions makes use of open disks centered at the points on the domain and range side, where the open disk is the interior region bounded by a circle centered at the point. Which other geometric shapes can we use instead of a circle of specified radius centered at the point? Please see Options (D) and (E) before answering and make the most appropriate selection. *Last time:* 12/22 *correct.*
(A) A square of specified side length centered at the point
(B) An equilateral triangle of specified side length centered at the point
(C) A regular hexagon of specified side length centered at the point
(D) Any of the above
(E) None of the above

Your answer: ________________________

(3) Here's a quick recap of the limit definition for a function of a vector variable. We say that $\lim_{\mathbf{x} \to \mathbf{c}} f(\mathbf{x}) = L$ if for every $\varepsilon > 0$ there exists $\delta > 0$ such that for all $\mathbf{x}$ satisfying $0 < |\mathbf{x} - \mathbf{c}| < \delta$, we have $|f(\mathbf{x}) - L| < \varepsilon$. We define $|\mathbf{x} - \mathbf{c}|$ as the Euclidean norm of $\mathbf{x} - \mathbf{c}$ where the Euclidean norm of a vector is the square root of the sum of the squares of its coordinates.

We could replace the Euclidean norm by other measurements. For instance, we could use:
(i) The *sum* of the absolute values of the coordinates of $\mathbf{x} - \mathbf{c}$.
(ii) The *maximum* of the absolute values of the coordinates of $\mathbf{x} - \mathbf{c}$.
(iii) The *minimum* of the absolute values of the coordinates of $\mathbf{x} - \mathbf{c}$.

For any of (i) - (iii), we could replace $|\mathbf{x} - \mathbf{c}|$ in our current definition of limit with that notion. The question is: for which of the replacements will our new notion of limit be the same as the old one? The deeper idea here is that limit depends upon a concept of what it means for two points to be close. So another way of phrasing the question is: which of the notions (i)-(iii) capture the same notion of closeness as the usual Euclidean distance?
(A) All of (i), (ii), and (iii).
(B) (i) and (ii) but not (iii).
(C) (i) and (iii) but not (ii).
(D) Only (i).
(E) None of (i), (ii), or (iii).

Your answer: ______________________________

(4) Suppose f is a function of two variables x, y and is defined on the whole xy-plane. Consider three conditions: (i) f is continuous on the whole xy-plane, (ii) for every fixed value $x = x_0$, the function $y \mapsto f(x_0, y)$ is continuous in y for all $y \in \mathbb{R}$, (iii) for every fixed value $y = y_0$, the function $x \mapsto f(x, y_0)$ is continuous in x for all $x \in \mathbb{R}$, (iv) the function $t \mapsto f(p(t), q(t))$ is continuous for all $t \in \mathbb{R}$ whenever p and q are both constant or linear functions (in other words, the restriction of f to any straight line in $\mathbb{R}^2$ is continuous).

Which of the following correctly describes the implications between (i), (ii), (iii), and (iv)?

(A) (i) implies both (ii) and (iii), and (ii) and (iii) together imply (iv).
(B) (i) implies (iv), and (iv) implies both (ii) and (iii).
(C) (iv) implies (ii) and (iii), and (ii) and (iii) together imply (i).
(D) (iv) implies (i), and (i) implies both (ii) and (iii).
(E) (ii) and (iii) together imply (iv), and (iv) implies (i).

Your answer: ______________________________