

CLASS QUIZ: FRIDAY FEBRUARY 1: MULTIVARIABLE FUNCTION BASICS

MATH 195, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): \_\_\_\_\_

PLEASE DISCUSS *ONLY* THE STARRED OR DOUBLE-STARRED QUESTIONS.

- (1) Suppose  $f$  is a function of two variables, defined on all of  $\mathbb{R}^2$ , with the property that  $f(x, y) = f(y, x)$  for all real numbers  $x$  and  $y$ . What does this say about the symmetry of the graph  $z = f(x, y)$  of  $f$ ? *Last time: 16/21 correct – DO NOT DISCUSS.*
- (A) It has mirror symmetry about the plane  $z = x + y$ .
  - (B) It has mirror symmetry about the plane  $x = y$ .
  - (C) It has mirror symmetry about the plane  $z = x - y$ .
  - (D) It has half turn symmetry about the line  $x = y = z$ .
  - (E) It has half turn symmetry about the origin.

Your answer: \_\_\_\_\_

- (2) (\*\*) Consider the function  $f(x, y) := ax + by$  where  $a$  and  $b$  are fixed nonzero reals. The level curves for this function are a bunch of parallel lines. What vector are they all parallel to? *Last time: 5/21 correct*
- (A)  $\langle a, b \rangle$
  - (B)  $\langle a, -b \rangle$ .
  - (C)  $\langle b, a \rangle$
  - (D)  $\langle b, -a \rangle$
  - (E)  $\langle a - b, a + b \rangle$

Your answer: \_\_\_\_\_

- (3) (\*\*) Suppose  $f$  is a function of one variable and  $g$  is a function of two variables. What is the relationship between the level curves of  $f \circ g$  and the level curves of  $g$ ? *Last time: 7/21 correct*
- (A) Each level curve of  $f \circ g$  is a union of level curves of  $g$  corresponding to the pre-images of the point under  $f$ .
  - (B) Each level curve of  $f \circ g$  is an intersection of level curves of  $g$  corresponding to the pre-images of the point under  $f$ .
  - (C) The level curves of  $f \circ g$  are precisely the same as the level curves of  $g$ .
  - (D) Each level curve of  $g$  is a union of level curves of  $f \circ g$ .
  - (E) Each level curve of  $g$  is an intersection of level curves of  $f \circ g$ .

Your answer: \_\_\_\_\_

- (4) Consider the following function  $f$  from  $\mathbb{R}^2$  to  $\mathbb{R}^2$ : the function that sends  $\langle x, y \rangle$  to  $\langle \frac{x+y}{2}, \frac{x-y}{2} \rangle$ . What is the image of  $\langle x, y \rangle$  under  $f \circ f$ ? *Last time: 12/21 correct – DO NOT DISCUSS.*
- (A)  $\langle x, y \rangle$
  - (B)  $\langle 2x, 2y \rangle$
  - (C)  $\langle x/2, y/2 \rangle$

- (D)  $\langle x + (y/2), y + (x/2) \rangle$
- (E)  $\langle 2x + y, 2x - y \rangle$

Your answer: \_\_\_\_\_

- (5) (\*\*) Consider the following functions defined on the subset  $x > 0$  of the  $xy$ -plane:  $f(x, y) = x^y$ . Consider the surface  $z = f(x, y)$ . What do the intersections of this surface with planes parallel to the  $xz$ -plane and  $yz$ -plane look like (ignore the following two special intersections: intersection with the plane  $x = 1$  and intersection with the plane  $y = 0$ , also ignore intersections that turn out to be empty). *Last time: 5/21 correct*
- (A) Intersections with any plane parallel to the  $xz$  or  $yz$  plane look like graphs of exponential functions.
  - (B) Intersections with any plane parallel to the  $xz$  or  $yz$  plane look like graphs of power functions (only positive inputs allowed).
  - (C) Intersections with any plane parallel to the  $xz$ -plane look like graphs of exponential functions, and intersections with any plane parallel to the  $yz$ -plane look like graphs of power functions (only positive inputs allowed).
  - (D) Intersections with any plane parallel to the  $yz$ -plane look like graphs of exponential functions, and intersections with any plane parallel to the  $xz$ -plane look like graphs of power functions (only positive inputs allowed).
  - (E) All the intersections are straight lines.

Your answer: \_\_\_\_\_