# TAKE-HOME CLASS QUIZ: DUE FRIDAY JANUARY 25: LIMITS, CONTINUITY, DIFFERENTIATION REVIEW

MATH 195, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ________________________________________

These questions are all related to single variable calculus. Specifically, these are some of the harder questions from material typically covered in Math 151/152. I've given these questions in past quizzes/tests in Math 152 and Math 153 and the scores indicated here are the scores in appearances of these questions in previous quizzes/tests.

**PLEASE FEEL FREE TO DISCUSS THESE QUESTIONS WITH OTHERS, BUT YOUR FINAL ANSWERS SHOULD BE THE ANSWERS YOU ARE PERSONALLY CONVINCED ABOUT**.

(1) Which of the following statements is **always true**? *Earlier scores:* 2/11*,* 9/16*,* 16/28
   (A) The range of a continuous nonconstant function on an open bounded interval (i.e., an interval of the form $(a, b)$) is an open bounded interval (i.e., an interval of the form $(m, M)$).
   (B) The range of a continuous nonconstant function on a closed bounded interval (i.e., an interval of the form $[a, b]$) is a closed bounded interval (i.e., an interval of the form $[m, M]$).
   (C) The range of a continuous nonconstant function on an open interval that may be bounded or unbounded (i.e., an interval of the form $(a, b)$,$(a, \infty)$, $(-\infty, a)$, or $(-\infty, \infty)$), is also an open interval that may be bounded or unbounded.
   (D) The range of a continuous nonconstant function on a closed interval that may be bounded or unbounded (i.e., an interval of the form $[a, b]$, $[a, \infty)$, $(-\infty, a]$, or $(-\infty, \infty)$) is also a closed interval that may be bounded or unbounded.
   (E) None of the above.

   Your answer: ____________________________

(2) For which of the following specifications is there **no continuous function** satisfying the specifications? *Earlier score:* 7/14*,* 21/28
   (A) Domain $(0, 1)$ and range $(0, 1)$
   (B) Domain $[0, 1]$ and range $(0, 1)$
   (C) Domain $(0, 1)$ and range $[0, 1]$
   (D) Domain $[0, 1]$ and range $[0, 1]$
   (E) None of the above, i.e., we can get a continuous function for each of the specifications.

   Your answer: ____________________________

(3) Suppose $f$ is a continuously differentiable function from the open interval $(0, 1)$ to $\mathbb{R}$. Suppose, further, that there are exactly 14 values of $c$ in $(0, 1)$ for which $f(c) = 0$. What can we say is **definitely true** about the number of values of $c$ in the open interval $(0, 1)$ for which $f'(c) = 0$? *Earlier scores:* 7/15*,* 19/28
   (A) It is at least 13 and at most 15.
   (B) It is at least 13, but we cannot put any upper bound on it based on the given information.
   (C) It is at most 15, but we cannot put any lower bound (other than the meaningless bound of 0) based on the given information.
   (D) It is at most 13.
   (E) It is at least 15.

   Your answer: ____________________________

(4) Consider the function $f(x) := \begin{cases} x, & 0 \leq x \leq 1/2 \\ x - (1/7), & 1/2 < x \leq 1 \end{cases}$. Define by $f^{[n]}$ the function obtained by iterating $f$ $n$ times, i.e., the function $f \circ f \circ f \circ \cdots \circ f$ where $f$ occurs $n$ times. What is the smallest $n$ for which $f^{[n]} = f^{[n+1]}$? *Earlier scores:* 3/16, 10/28

(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Your answer: ____________________________

(5) Suppose $f$ and $g$ are functions $(0,1)$ to $(0,1)$ that are both right continuous on $(0,1)$. Which of the following is *not* guaranteed to be right continuous on $(0,1)$? *Earlier scores:* 3/11, 9/14, 20/28

(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
(B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$
(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(E) None of the above, i.e., they are all guaranteed to be right continuous functions

Your answer: ____________________________

(6) Suppose $f$ and $g$ are increasing functions from $\mathbb{R}$ to $\mathbb{R}$. Which of the following functions is *not* guaranteed to be an increasing function from $\mathbb{R}$ to $\mathbb{R}$? *Earlier scores:* 1/15, 9/16, 18/28

(A) $f + g$
(B) $f \cdot g$
(C) $f \circ g$
(D) All of the above, i.e., none of them is guaranteed to be increasing.
(E) None of the above, i.e., they are all guaranteed to be increasing.

Your answer: ____________________________

(7) Suppose $F$ and $G$ are two functions defined on $\mathbb{R}$ and $k$ is a natural number such that the $k^{th}$ derivatives of $F$ and $G$ exist and are equal on all of $\mathbb{R}$. Then, $F - G$ must be a polynomial function. What is the **maximum possible degree** of $F - G$? (Note: Assume constant polynomials to have degree zero) *Earlier scores:* 6/16, 10/28

(A) $k - 2$
(B) $k - 1$
(C) $k$
(D) $k + 1$
(E) There is no bound in terms of $k$.

Your answer: ____________________________

(8) Suppose $f$ is a continuous function on $\mathbb{R}$. Clearly, $f$ has antiderivatives on $\mathbb{R}$. For all but one of the following conditions, it is possible to guarantee, without any further information about $f$, that there exists an antiderivative $F$ satisfying that condition. **Identify the exceptional condition** (i.e., the condition that it may not always be possible to satisfy). *Earlier scores:* 3/16, 10/28

(A) $F(1) = F(0)$.
(B) $F(1) + F(0) = 0$.
(C) $F(1) + F(0) = 1$.
(D) $F(1) = 2F(0)$.
(E) $F(1)F(0) = 0$.

Your answer: ____________________________

(9) Suppose $F$ is a function defined on $\mathbb{R} \setminus \{0\}$ such that $F'(x) = -1/x^2$ for all $x \in \mathbb{R} \setminus \{0\}$. Which of the following pieces of information is/are **sufficient** to determine $F$ completely? Please see options (D) and (E) before answering. *Earlier scores:* 4/16, 15/28

(A) The value of $F$ at any two positive numbers.

(B) The value of $F$ at any two negative numbers.
(C) The value of $F$ at a positive number and a negative number.
(D) Any of the above pieces of information is sufficient, i.e., we need to know the value of $F$ at any two numbers.
(E) None of the above pieces of information is sufficient.

Your answer: ____________________________

(10) Suppose $F$ and $G$ are continuously differentiable functions on all of $\mathbb{R}$ (i.e., both $F'$ and $G'$ are continuous). Which of the following is **not necessarily true**? *Earlier scores:* 0, 10/16, 11/28

(A) If $F'(x) = G'(x)$ for all integers $x$, then $F - G$ is a constant function when restricted to integers, i.e., it takes the same value at all integers.
(B) If $F'(x) = G'(x)$ for all numbers $x$ that are not integers, then $F - G$ is a constant function when restricted to the set of numbers $x$ that are not integers.
(C) If $F'(x) = G'(x)$ for all rational numbers $x$, then $F - G$ is a constant function when restricted to the set of rational numbers.
(D) If $F'(x) = G'(x)$ for all irrational numbers $x$, then $F - G$ is a constant function when restricted to the set of irrational numbers.
(E) None of the above, i.e., they are all necessarily true.

Your answer: ____________________________