# TAKE-HOME CLASS QUIZ: DUE WEDNESDAY JANUARY 23: VECTORS, 3D, AND PARAMETRIC STUFF – MISCELLANEA

MATH 195, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

**THIS IS A TAKE-HOME CLASS QUIZ, BUT I WILL GIVE YOU ABOUT 5 MINUTES TO REVIEW YOUR ANSWERS IN CLASS AND DISCUSS WITH OTHER STUDENTS.**

**YOU ARE FREE TO DISCUSS *ALL* QUESTIONS, BUT PLEASE FINALLY ENTER ONLY THE ANSWER OPTION YOU ARE PERSONALLY MOST CONVINCED ABOUT – DON'T ENGAGE IN GROUPTHINK.**

Yes, you are free to discuss *all* questions for this quiz.

(1) Suppose we are given three subsets $\Gamma_1$, $\Gamma_2$, and $\Gamma_3$ of $\mathbb{R}^3$ where $\Gamma_1$ is the set of solutions to $F_1(x,y,z) = 0$, $\Gamma_2$ is the set of solutions to $F_2(x,y,z) = 0$, and $\Gamma_3$ is the set of solutions to $F_3(x,y,z) = 0$. Which of the following equations gives precisely the set of points that lie in *at least two* of the subsets $\Gamma_1$, $\Gamma_2$, and $\Gamma_3$?
(A) $F_1(x,y,z)F_2(x,y,z)F_3(x,y,z) = 0$
(B) $(F_1(x,y,z))^2 + (F_2(x,y,z))^2 + (F_3(x,y,z))^2 = 0$
(C) $(F_1(x,y,z) + F_2(x,y,z) + F_3(x,y,z))^2 = 0$
(D) $(F_1(x,y,z)F_2(x,y,z)) + (F_2(x,y,z)F_3(x,y,z)) + (F_3(x,y,z)F_1(x,y,z)) = 0$
(E) $(F_1(x,y,z)F_2(x,y,z))^2 + (F_2(x,y,z)F_3(x,y,z))^2 + (F_3(x,y,z)F_1(x,y,z))^2 = 0$

Your answer: ______________________

(2) Suppose we are given three subsets $\Gamma_1$, $\Gamma_2$, and $\Gamma_3$ of $\mathbb{R}^3$ where $\Gamma_1$ is the set of solutions to $F_1(x,y,z) = 0$, $\Gamma_2$ is the set of solutions to $F_2(x,y,z) = 0$, and $\Gamma_3$ is the set of solutions to $F_3(x,y,z) = 0$. Which of the following equations gives precisely the set $\Gamma_1 \cap (\Gamma_2 \cup \Gamma_3)$?
(A) $F_1(x,y,z) + F_2(x,y,z)F_3(x,y,z) = 0$
(B) $(F_1(x,y,z))^2 + (F_2(x,y,z)F_3(x,y,z))^2 = 0$
(C) $(F_1(x,y,z) + F_2(x,y,z)F_3(x,y,z))^2 = 0$
(D) $F_1(x,y,z)(F_2(x,y,z) + F_3(x,y,z))^2 = 0$
(E) $F_1(x,y,z)((F_2(x,y,z))^2 + (F_3(x,y,z))^2) = 0$

Your answer: ______________________

(3) Suppose we are given three subsets $\Gamma_1$, $\Gamma_2$, and $\Gamma_3$ of $\mathbb{R}^3$ where $\Gamma_1$ is the set of solutions to $F_1(x,y,z) = 0$, $\Gamma_2$ is the set of solutions to $F_2(x,y,z) = 0$, and $\Gamma_3$ is the set of solutions to $F_3(x,y,z) = 0$. Which of the following equations gives precisely the set $\Gamma_1 \cup (\Gamma_2 \cap \Gamma_3)$?
(A) $F_1(x,y,z) + F_2(x,y,z)F_3(x,y,z) = 0$
(B) $(F_1(x,y,z))^2 + (F_2(x,y,z)F_3(x,y,z))^2 = 0$
(C) $(F_1(x,y,z) + F_2(x,y,z)F_3(x,y,z))^2 = 0$
(D) $F_1(x,y,z)(F_2(x,y,z) + F_3(x,y,z))^2 = 0$
(E) $F_1(x,y,z)((F_2(x,y,z))^2 + (F_3(x,y,z))^2) = 0$

Your answer: ______________________

(4) Start with two vectors $a$ and $b$ in $\mathbb{R}^3$ such that $a \times b \neq 0$. Consider a sequence of vectors $c_1, c_2, \dots, c_n, \dots$ in $\mathbb{R}^3$ (note: each $c_n$ is a three-dimensional vector) defined as follows: $c_1 = a \times b$ and $c_{n+1} = a \times c_n$ for $n \geq 1$. Which *one* of the following statements is **false** about the $c_n$s? (5 points)

(A) All the vectors $c_n$ are nonzero vectors.
(B) $c_n$ and $c_{n+1}$ are orthogonal for every $n$.
(C) $c_n$ and $c_{n+2}$ are parallel for every $n$.
(D) $c_n$ and $a$ are orthogonal for every $n$.
(E) $c_n$ and $b$ are orthogonal for every $n$.

Your answer: ____________________________

(5) As a general rule, what would you expect should be the dimensionality of the set of solutions to $m$ independent and consistent equations in $n$ variables? By solution, we mean here that a $n$-tuple with coordinates in $\mathbb{R}$ (in other words, a point in $\mathbb{R}^n$) that satisfy all the $m$ equations. Assume $n \geq m \geq 1$.
(A) $n$
(B) $m$
(C) $n - 1$
(D) $n - m$
(E) 1

Your answer: ____________________________

(6) As a general rule, what would you expect should be the dimensionality of the set of points in $\mathbb{R}^n$ that satisfy at least one of $m$ independent and consistent equations in $n$ variables? Assume $n \geq m \geq 1$.
(A) $n$
(B) $m$
(C) $n - 1$
(D) $n - m$
(E) 1

Your answer: ____________________________

(7) Measuring time $t$ in seconds since the beginning of the year 2013, and stock prices on a $24 \times 7$ stock exchange in predetermined units, the stock prices of companies $A$, $B$, and $C$ were found to be given by $30 + t/5000000 - \sin(t/10000)$, $16 + 7t/3000000$, and $40 + t/1000000 - 5\sin(t/10000)$. To what extent can we deduce the stock prices of the companies from each other at a given point in time, without knowing what the time is?
(A) The stock price of any of the three companies can be used to deduce the other stock prices.
(B) The stock price of company $A$ can be used to deduce the stock prices of companies $B$ and $C$, but no other deductions are possible.
(C) The stock price of company $A$ can be used to deduce the stock prices of companies $B$ and $C$, and the stock price of company $C$ can be used to deduce the stock prices of companies $A$ and $B$.
(D) The stock price of company $B$ can be used to determine the stock prices of companies $A$ and $C$, and no other deductions are possible.
(E) The stock price of company $B$ can be used to determine the stock prices of companies $A$ and $C$, and the stock prices of companies $A$ and $C$ can be used to deduce each other but cannot be used to uniquely deduce the stock price of company $B$.

Your answer: ____________________________

(8) Lushanna is coaching 30 young athletes for a 100 meter sprint. Every day, at the beginning of the day, she asks the athlete to run 100 meters as fast as they can and notes the time taken. She thus gets a vector with 30 coordinates (measuring the time taken by all the athletes) everyday. Lushanna then plots a graph in thirty-dimensional space that includes all the points for her daily measurements. Each of the following is a sign that Lushanna's young athletes are improving. Which of these signs is **strongest**, in the sense that it would imply all the others?
(A) The norm (length) of the vector every day (after the first) is less than the norm of the vector the previous day.

(B) The sum of the coordinates of the vector every day (after the first) is less than the sum of the coordinates of the vector the previous day.
(C) The minimum of the coordinates of the vector every day (after the first) is less than the minimum of the coordinates of the vector the previous day.
(D) The maximum of the coordinates of the vector every day (after the first) is less than the maximum of the coordinates of the vector the previous day.
(E) Each of the coordinates of the vector every day (after the first) is less than the corresponding coordinate of the vector the previous day.

Your answer: ____________________________

(9) In a closed system (no mass exchanged with the surroundings) a reversible chemical reaction $A+B \to C+D$, and its reverse, are proceeding. There are no other chemicals in the system, and no other reactions are proceeding in the system. A chemist studying the reaction decides to track the masses of $A$, $B$, $C$, and $D$ in the system as a function of time, and plots a parametric curve in four-dimensional space. What can we say about the nature of the curve, ignoring the parametrization (i.e., just looking at the set of points covered)?
(A) It is a part of a straight line.
(B) It is a part of a circle.
(C) It is a part of a parabola.
(D) It is a part of an astroid.
(E) It is a part of a cissoid.

Your answer: ____________________________

*Lobbying special*: Casa is a lobbyist for a special interest group. There are three politicians $P_1, P_2, P_3$ competing for a general election. Casa has computed that the probabilities of the politicians winning are $p_1$ for $P_1$, $p_2$ for $P_2$, and $p_3$ for $P_3$, with $p_1, p_2, p_3 \in [0,1]$ and $p_1 + p_2 + p_3 = 1$. Casa estimates a payoff of $m_1$ money units to her special interest group if $P_1$ wins, $m_2$ money units if $P_2$ wins, and $m_3$ money units if $P_3$ wins. (These payoffs may be in terms of passage of favorable laws, repeal of unfavorable laws, or enforcement of laws unfavorable to competitors).

(10) What is the expected payoff to the special interest group that Casa represents?
(A) $m_1 + m_2 + m_3$
(B) $(m_1 + m_2 + m_3)/3$
(C) $(p_1 + p_2 + p_3)(m_1 + m_2 + m_3)$
(D) $p_1 m_1 + p_2 m_2 + p_3 m_3$
(E) $\sqrt{m_1^2 + m_2^2 + m_3^2}$

Your answer: ____________________________

(11) Casa has discovered that the bribe multipliers of the politicians are the positive reals $b_1$, $b_2$, and $b_3$ respectively. In other words, if Casa donates $u_i$ money units to $P_i$, then the expected payoff from politician $P_i$ winning is now $m_i + b_i u_i$. Consider the vectors $p = \langle p_1, p_2, p_3 \rangle$, $m = \langle m_1, m_2, m_3 \rangle$, $c = \langle p_1 b_1, p_2 b_2, p_3 b_3 \rangle$, and $f = \langle p_1/b_1, p_2/b_2, p_3/b_3 \rangle$ and let $u = \langle u_1, u_2, u_3 \rangle$ be the vector of the bribe quantities Casa gives to the politicians respectively. Assume that bribing politicians does not affect the relative probabilities of winning the election. Which of the following describes Casa's expected payoff from the election, once the bribe is made (if you want to include the cost of bribes, you'd need to subtract $u_1 + u_2 + u_3$ from this answer, but we're not doing that. *Note: Some of the answer options may not make sense from a dimensions/units viewpoint, but the correct answer does make sense.*
(A) $p \cdot (m + u)$
(B) $p \cdot (m + (b \cdot u))$
(C) $p \cdot (m + (f \cdot u))$
(D) $(p \cdot m) + (c \cdot u)$
(E) $p \cdot (f \cdot m + u)$

Your answer: ____________________________

(12) Continuing with the full setup of the preceing question, what is Casa's optimal bribing strategy on a fixed budget of money to be used for bribes?

(A) Donate all the money to the politician with the maximum value of $p_i b_i$, i.e., to the politician corresponding to the largest coordinate of the vector $c$.

(B) Donate all the money to the politician with the minimum value of $p_i b_i$, i.e., to the politician corresponding to the smallest coordinate of the vector $c$.

(C) Donate all the money to the politician with the maximum value of $p_i/b_i$, i.e., to the politician corresponding to the largest coordinate of the vector $f$.

(D) Donate all the money to the politician with the minimum value of $p_i/b_i$, i.e., to the politician corresponding to the smallest coordinate of the vector $f$.

(E) Split the bribery budget between the politicians in the ratio $p_1 b_1 : p_2 b_2 : p_3 b_3$.

Your answer: ____________________________