CLASS QUIZ: FRIDAY JANUARY 18: VECTORS

MATH 195, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

YOU ARE ALLOWED TO DISCUSS ONLY QUESTIONS THAT BEGIN WITH A (*) OR (**). PLEASE ATTEMPT ALL OTHER QUESTIONS BY YOURSELF. EVEN FOR THE QUESTIONS YOU DISCUSS, PLEASE FINALLY ENTER ONLY THE ANSWER OPTION YOU ARE PERSONALLY MOST CONVINCED ABOUT – DON'T ENGAGE IN GROUPTHINK.

(1) Suppose S is a collection of *nonzero* vectors in $\mathbb{R}^3$ with the property that the dot product of any two distinct elements of S is zero. What is the maximum possible size of S? *Last time:* 14/23 *correct*
(A) 1
(B) 2
(C) 3
(D) 4
(E) There is no finite bound on the size of S

Your answer: ____________________________

(2) Suppose S is a collection of *nonzero* vectors in $\mathbb{R}^3$ such that the cross product of any two distinct elements of S is the zero vector. What is the maximum possible size of S? *Last time:* 17/23 *correct*
(A) 1
(B) 2
(C) 3
(D) 4
(E) There is no finite bound on the size of S

Your answer: ____________________________

(3) (**) Suppose a and b are vectors in $\mathbb{R}^3$. Which of the following is/are true? *Last time:* 6/23 *correct*
(A) If both a and b are nonzero vectors, then $a \times b$ is a nonzero vector.
(B) If $a \times b$ is a nonzero vector, then $a \cdot (a \times b)$ is a nonzero real number.
(C) If $a \times b$ is a nonzero vector, then $a \times (a \times b)$ is a nonzero vector.
(D) All of the above
(E) None of the above

Your answer: ____________________________

(4) (*) Suppose a, b, c, and d are vectors in $\mathbb{R}^3$, with $a \times b \neq 0$ and $c \times d \neq 0$. What does $(a \times b) \times (c \times d) = 0$ mean? *Last time:* 9/23 *correct*

(A) Both the vectors a and b are perpendicular to both the vectors c and d.
(B) a and b are perpendicular to each other and c and d are perpendicular to each other.
(C) a and c are perpendicular to each other and b and d are perpendicular to each other.
(D) The plane spanned by a and b is perpendicular to the plane spanned by c and d.
(E) a, b, c, and d are all coplanar.

Your answer: ____________________________

(5) (**) The *correlation* between two vectors in $\mathbb{R}^n$ is defined as the quotient of the dot product of the vectors by the product of their lengths. Suppose a, b, and c are vectors in $\mathbb{R}^n$ such that the correlation between vectors a and b is a number x and the correlation between b and c is a number y, and suppose x, y are both positive. What is the maximum possible value of the correlation between a and c given this information? *Hint: Geometrically if θ_{ab} is the angle between a and b, θ_{bc} is the angle between b and c, and θ_{ac} is the angle between a and c, then $|\theta_{ab} - \theta_{bc}| \leq \theta_{ac} \leq \theta_{ab} + \theta_{bc}$. Last time:* 5/23 *correct*
(A) xy
(B) $\max\{1, xy\}$
(C) $\min\{1, xy\}$
(D) $xy + \sqrt{(1-x^2)(1-y^2)}$
(E) $xy - \sqrt{(1-x^2)(1-y^2)}$

Your answer: ____________________________

(6) If the correlation between nonzero vector v and nonzero vector w in $\mathbb{R}^n$ is c, then we say that the *proportion* of vector w *explained by* vector v is c^2. If $v_1, v_2, \ldots, v_k$ are all pairwise orthogonal nonzero vectors, and c_i is the correlation between v_i and w, then $c_1^2 + c_2^2 + \cdots + c_k^2 \leq 1$, with equality occurring if and only if $k = n$. (This is all a result of the Pythagorean theorem). If $k < n$, then $1 - (c_1^2 + c_2^2 + \cdots + c_k^2)$ is the *unexplained proportion* of w.

Suppose w is the *variation of beauty* vector, v_1 is the *variation of genes* vector, and v_2 is the *variance of make-up* vector. Assume that v_1 and v_2 are orthogonal (i.e., there is no correlation between genes and make-up choice). If the correlation between v_1 and w is 0.6 and the correlation between v_2 and w is 0.3, what proportion of the variation of beauty remains unexplained (i.e., is not explained by either genes or make-up)? *Last time:* 17/23 *correct*
(A) 0.1
(B) 0.19
(C) 0.55
(D) 0.74
(E) 1

Your answer: ____________________________