TAKE-HOME CLASS QUIZ: DUE OCTOBER 10: LEAST UPPER BOUND AXIOM

MATH 153, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

PLEASE DO *NOT* DISCUSS THESE QUESTIONS. You are, however, free to discuss the relevant material without specifically referring to the questions.

This is, hopefully, a fairly easy quiz by the standards that you are hopefully accustomed to by now, and you should be able to do all the questions correctly.

(1) Which of the following is an **alternative characterization** of the least upper bound of a nonempty subset S of $\mathbb{R}$ that is bounded from above?
- (A) It is the greatest lower bound of the set of all upper bounds (in $\mathbb{R}$) of S.
- (B) It is the least upper bound of the set of all upper bounds (in $\mathbb{R}$) of S.
- (C) It is the greatest lower bound of the set of all lower bounds (in $\mathbb{R}$) of S.
- (D) It is the least upper bound of the set of all lower bounds (in $\mathbb{R}$) of S.
- (E) None of the above.

Your answer: ____________________________

(2) Consider the following four intervals where $a < b$ are both fixed real numbers: the open interval (a, b), the closed interval $[a, b]$, the left-closed right-open interval $[a, b)$, and the left-open right-closed interval $(a, b]$. Which of the following is **true** about the upper and lower bounds of these intervals?
- (A) All four intervals have the same greatest lower bound as each other. Also, all four intervals have the same least upper bound as each other.
- (B) All four intervals have greatest lower bounds and least upper bounds. However, the greatest lower bounds for $[a, b]$ and $[a, b)$, while equal to each other, differ from the greatest lower bound for $(a, b]$ and (a, b), which in turn are equal to each other. Similarly, the least upper bounds for $[a, b]$ and $(a, b]$, while equal to each other, differ from the least upper bound for $[a, b)$ and (a, b), which in turn are equal to each other.
- (C) The intervals $[a, b]$ and $[a, b)$ have a greatest lower bound and the intervals $(a, b]$ and (a, b) do not. Further, the intervals $[a, b]$ and $(a, b]$ have a least upper bound, and the intervals $[a, b)$ and (a, b) do not.
- (D) None of the intervals has a greatest lower bound or a least upper bound.
- (E) $[a, b]$ is the only interval among the four intervals that has a greatest lower bound. It is also the only interval that has a least upper bound.

Your answer: ____________________________

(3) The greatest lower bound of a sequence is defined as the greatest lower bound of the set of values that it takes (i.e., its range as a function). Similarly, we can define the least upper bound of a sequence. Which of the following is **true**?
- (A) Any bounded monotonic sequence converges to its greatest lower bound.
- (B) Any bounded monotonic sequence converges to its least upper bound.
- (C) Any bounded monotonic sequence is convergent. It converges to its least upper bound if it is non-increasing (i.e., weakly decreasing) and to its greatest lower bound if it is non-decreasing (i.e., weakly increasing).
- (D) Any bounded monotonic sequence is convergent. It converges to its greatest lower bound if it is non-increasing (i.e., weakly decreasing) and to its least upper bound if it is non-decreasing (i.e., weaky increasing).
- (E) None of the above.

Your answer: ____________________________

(4) Building on the definition of greatest lower bound and least upper bound of a sequence in the previous question, which of the following is **true**?
  (A) A sequence is non-decreasing (i.e., weakly increasing) if and only if its first term equals its greatest lower bound.
  (B) If a sequence is non-decreasing (i.e., weakly increasing), then its first term is its greatest lower bound, but the converse is not true in general.
  (C) If the first term of a sequence is its greatest lower bound, then the sequence is non-decreasing (i.e., weakly increasing), but the converse is not true in general.
  (D) All of the above.
  (E) None of the above.

Your answer: ____________________________

(5) Suppose S is a nonempty bounded subset of $\mathbb{R}$, so that it has a finite greatest lower bound and a finite least upper bound. Denote by $-S$ the set $\{-s : s \in S\}$, i.e., the set of negatives of elements of S. Which of the following is **true** about $-S$?
  (A) The least upper bound of $-S$ equals the least upper bound of S, and the greatest lower bound of $-S$ equals the greatest lower bound of S.
  (B) The least upper bound of $-S$ equals the greatest lower bound of S, and the greatest lower bound of $-S$ equals the least upper bound of S.
  (C) The least upper bound of $-S$ equals the negative of the least upper bound of S, and the greatest lower bound of $-S$ equals the negative of the greatest lower bound of S.
  (D) The least upper bound of $-S$ equals the negative of the greatest lower bound of S, and the greatest lower bound of $-S$ equals the negative of the least upper bound of S.
  (E) None of the above.

Your answer: ____________________________