CLASS QUIZ: JANUARY 18: MATHEMATICAL INDUCTION

MATH 153, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________________

For all these questions, *natural number* refers to positive integer. In particular, 0 is not considered to be a natural number.

(1) Suppose S is a subset of the natural numbers with the property that $1 \in S$ and $k \in S \implies k+2 \in S$. What can we conclude is **definitely true** about S? *Last year:* 18/29 *correct*
(A) S contains all natural numbers.
(B) S contains all natural numbers other than 2. It may or may not contain 2.
(C) S contains all odd natural numbers.
(D) S contains all even natural numbers.
(E) S does not contain any natural number other than 1.

Your answer: ____________________________

(2) Suppose S is a subset of the natural numbers with the property that whenever $k \in S$, we have $k+5 \in S$. Which of these is the **smallest subset** T with the property that checking $T \subseteq S$ is sufficient to show that S is the set of all natural numbers? *Last year:* 21/29 *correct*
(A) $\{1, 2, 3\}$
(B) $\{1, 2, 3, 4\}$
(C) $\{1, 2, 3, 4, 5\}$
(D) $\{1, 4\}$
(E) $\{1, 3, 5\}$

Your answer: ____________________________

(3) Consider the function $f(x) := a \sin x + b \cos x$, with a, b nonzero reals. The n^{th} derivative of f is denote $f^{(n)}$. The association $n \mapsto f^{(n)}$ is periodic, i.e., there is a unique smallest positive integer h such that $f^{(n+h)} = f^{(n)}$ for all n. What is **this value** of h? *Last year:* 25/29 *correct*
(A) 1
(B) 2
(C) 3
(D) 4
(E) 5

Your answer: ____________________________

(4) What is the correct **general expression** for the **sum** $\sum_{k=2}^{n} \frac{1}{k^2-1}$ for a positive integer $n \geq 2$? *Last year:* 18/29 *correct*
(A) $\frac{3}{2} - \frac{2n+3}{2(n+1)}$
(B) $\frac{3}{2} - \frac{2n+3}{n(n+1)}$
(C) $\frac{3}{4} - \frac{2n+1}{(n+1)(n+2)}$
(D) $\frac{3}{4} - \frac{2n-1}{2n(n-1)}$
(E) $\frac{3}{4} - \frac{2n+1}{2n(n+1)}$

Your answer: ____________________________