TAKE HOME CLASS QUIZ: TURN IN NOVEMBER 30: INTEGRATION

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

THIS IS A TAKE HOME QUIZ.

FEEL FREE TO DISCUSS ALL QUESTIONS, BUT ATTEMPT THEM YOURSELF FIRST.

(1) What is the limit $\lim_{x\to\infty}\left[\left(\int_0^x \sin^2\theta d\theta\right)/x\right]$? *Last year:* 13/16 *correct*
   (A) $1/2$
   (B) 1
   (C) $1/\pi$
   (D) $2/\pi$
   (E) $1/(2\pi)$

   Your answer: ____________________________

(2) Consider the substitution $u = -1/x$ for the integral $\int \frac{dx}{x^2+1}$. What is the **new integral**? *Last year:* 8/16 *correct*
   (A) $\int \frac{du}{u(u^2+1)}$
   (B) $\int \frac{du}{u^2+1}$
   (C) $\int \frac{udu}{u^2+1}$
   (D) $\int \frac{u^2du}{u^2+1}$
   (E) $\int \frac{u^2du}{(u^2+1)^2}$

   Your answer: ____________________________

(3) *Hard*: What is the **value** of $c \in (0,\infty)$ such that $\int_0^c \frac{dx}{x^2+1} = \lim_{a\to\infty}\int_c^a \frac{dx}{x^2+1}$? *Last year:* 8/16 *correct*
   (A) $\frac{1}{\sqrt{3}}$
   (B) $\frac{1}{\sqrt{2}}$
   (C) 1
   (D) $\sqrt{2}$
   (E) $\sqrt{3}$

   Your answer: ____________________________

(4) Suppose f is a continuous nonconstant even function on $\mathbb{R}$. Which of the following is **true**? *Last year:* 4/16 *correct*
   (A) Every antiderivative of f is an even function.
   (B) f has exactly one antiderivative that is an even function.
   (C) Every antiderivative of f is an odd function.

(D) f has exactly one antiderivative that is an odd function.
(E) None of the antiderivatives of f is either an even or an odd function.

Your answer: ____________________________

(5) Suppose f is a continuous nonconstant odd function on $\mathbb{R}$. Which of the following is **true**? *Last year:* 13/16 *correct*

(A) Every antiderivative of f is an even function.
(B) f has exactly one antiderivative that is an even function.
(C) Every antiderivative of f is an odd function.
(D) f has exactly one antiderivative that is an odd function.
(E) None of the antiderivatives of f is either an even or an odd function.

Your answer: ____________________________

(6) Suppose f is a continuous nonconstant periodic function on $\mathbb{R}$ with period h. Which of the following is **true**? *Last year:* 5/16 *correct*

(A) Every antiderivative of f is a periodic function with period h, regardless of the choice of f.
(B) For some choices of f, every antiderivative of f is a periodic function; for all others, f has no periodic antiderivative.
(C) f has exactly one periodic antiderivative for every choice of f.
(D) For some choices of f, f has exactly one periodic antiderivative; for all others, f has no periodic antiderivative.
(E) Regardless of the choice of f, no antiderivative of f can be periodic.

Your answer: ____________________________

(7) Consider a continuous increasing function f defined on the nonnegative real numbers. Define $m_f(a)$, for $a > 0$, as the unique value $c \in [0, a]$ such that $f(c)$ is the mean value of f on the interval $[0, a]$.

If $f(x) := x^n$, n an integer greater than 1, what kind of function is m_f (your answer should be valid for all n)? *Last year:* 1/16 *correct*

(A) $m_f(a)$ is a constant λ dependent on n but independent of a.
(B) It is a function of the form $m_f(a) = \lambda a$, where λ is a constant depending on n.
(C) It is a function of the form $m_f(a) = \lambda a^{n-1}$, where λ is a constant depending on n.
(D) It is a function of the form $m_f(a) = \lambda a^n$, where λ is a constant depending on n.
(E) It is a function of the form $m_f(a) = \lambda a^{n+1}$, where λ is a constant depending on n.

Your answer: ____________________________