CLASS QUIZ: NOVEMBER 23: MEMORY LANE

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) For which of the following specifications is there **no continuous function** satisfying the specifications?
(A) Domain $[0, 1]$ and range $[0, 1]$
(B) Domain $[0, 1]$ and range $(0, 1)$
(C) Domain $(0, 1)$ and range $[0, 1]$
(D) Domain $(0, 1)$ and range $(0, 1)$
(E) None of the above, i.e., we can get a continuous function for each of the specifications.

Your answer: ____________________________

(2) Suppose f and g are continuous functions on $\mathbb{R}$, such that f is continuously differentiable everywhere and g is continuously differentiable everywhere except at c, where it has a vertical tangent. What can we say is **definitely true** about $f \circ g$?
(A) It has a vertical tangent at c.
(B) It has a vertical cusp at c.
(C) It has either a vertical tangent or a vertical cusp at c.
(D) It has neither a vertical tangent nor a vertical cusp at c.
(E) We cannot say anything for certain.

Your answer: ____________________________

(3) Consider the function $p(x) := x^{2/3}(x-1)^{3/5} + (x-2)^{7/3}(x-5)^{4/3}(x-6)^{4/5}$. For what values of x does the graph of p have a vertical cusp at $(x, p(x))$?
(A) $x = 0$ only.
(B) $x = 0$ and $x = 5$ only.
(C) $x = 5$ and $x = 6$ only.
(D) $x = 0$ and $x = 6$ only.
(E) $x = 0$, $x = 5$, and $x = 6$.

Your answer: ____________________________

(4) Consider the function $f(x) := \left\{ \begin{array}{rl} x, & 0 \le x \le 1/2 \\ x^2, & 1/2 < x \le 1 \end{array} \right.$. What is $f \circ f$?

(A) $x \mapsto \left\{ \begin{array}{rl} x, & 0 \le x \le 1/2 \\ x^4, & 1/2 < x \le 1 \end{array} \right.$

(B) $x \mapsto \left\{ \begin{array}{rl} x, & 0 \le x \le 1/2 \\ x^2, & 1/2 < x \le 1 \end{array} \right.$

(C) $x \mapsto \begin{cases} x, & 0 \le x \le 1/2 \\ x^2, & 1/2 < x \le 1/\sqrt{2} \\ x^4, & 1/\sqrt{2} < x \le 1 \end{cases}$

(D) $x \mapsto \begin{cases} x, & 0 \le x \le 1/\sqrt{2} \\ x^2, & 1/\sqrt{2} < x \le 1 \end{cases}$

(E) $x \mapsto \begin{cases} x, & 0 \le x \le 1/\sqrt{2} \\ x^4, & 1/\sqrt{2} < x \le 1 \end{cases}$

Your answer: ______________________________

(5) Suppose f and g are functions $(0,1)$ to $(0,1)$ that are both right continuous on $(0,1)$. Which of the following is *not* guaranteed to be right continuous on $(0,1)$?

(A) $f+g$, i.e., the function $x \mapsto f(x)+g(x)$
(B) $f-g$, i.e., the function $x \mapsto f(x)-g(x)$
(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(E) None of the above, i.e., they are all guaranteed to be right continuous functions

Your answer: ______________________________

(6) For a partition $P = x_0 < x_1 < x_2 < \cdots < x_n$ of $[a,b]$ (with $x_0 = a$, $x_n = b$) define the norm $\|P\|$ as the maximum of the values $x_i - x_{i-1}$. Which of the following **is always true** for any continuous function f on $[a,b]$? (5 points)

(A) If P_1 is a finer partition than P_2, then $\|P_2\| \le \|P_1\|$ (Here, *finer* means that, as a set, $P_2 \subseteq P_1$, i.e., all the points of P_2 are also points of P_1).
(B) If $\|P_2\| \le \|P_1\|$, then $L_f(P_2) \le L_f(P_1)$ (where L_f is the lower sum).
(C) If $\|P_2\| \le \|P_1\|$, then $U_f(P_2) \le U_f(P_1)$ (where U_f is the upper sum).
(D) If $\|P_2\| \le \|P_1\|$, then $L_f(P_2) \le U_f(P_1)$.
(E) All of the above.

Your answer: ______________________________

(7) A disk of radius r in the xy-plane is translated parallel to itself with its center moving in the yz-plane along the semicircle $y^2+z^2 = R^2, y \ge 0$. The solid thus obtained can be thought of as a *cylinder of bent spine* with cross sections being disks of radius r along the xy-plane and the centers forming a semicircle of radius R in the yz-plane, with the z-value ranging from $-R$ to R. What is the volume of this solid?

(A) $2\pi r^2 R$
(B) $\pi^2 r^2 R$
(C) $2\pi r R^2$
(D) $\pi^2 r R^2$
(E) $\pi^2 R^3$

Your answer: ______________________________