CLASS QUIZ: NOVEMBER 21: ONE-ONE FUNCTIONS

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) For one of these function types for a continuous function from $\mathbb{R}$ to $\mathbb{R}$, it is *possible* to also be a one-to-one function. What is that function type? *Last year:* 15/15 *correct*
  (A) Function whose graph has mirror symmetry about a vertical line.
  (B) Function whose graph has half turn symmetry about a point on it.
  (C) Periodic function.
  (D) Function having a point of local minimum.
  (E) Function having a point of local maximum.

  Your answer: ____________________________

(2) (**) Suppose f, g, and h are continuous one-to-one functions whose domain and range are both $\mathbb{R}$. **What can we say** about the functions $f + g$, $f + h$, and $g + h$? *Last year:* 2/15 *correct*
  (A) They are all continuous one-to-one functions with domain $\mathbb{R}$ and range $\mathbb{R}$.
  (B) At least two of them are continuous one-to-one functions with domain $\mathbb{R}$ and range $\mathbb{R}$ – however, we cannot say more.
  (C) At least one of them is a continuous one-to-one function with domain $\mathbb{R}$ and range $\mathbb{R}$ – however, we cannot say more.
  (D) Either all three sums are continuous one-to-one functions whose domain and range are both $\mathbb{R}$, or none is.
  (E) It is possible that none of the sums is a continuous one-to-one function whose domain and range are both $\mathbb{R}$; it is also possible that one, two, or all the sums are continuous one-to-one functions whose domain and range are both $\mathbb{R}$.

  Your answer: ____________________________

(3) (**) Suppose f is a one-to-one function with domain a closed interval $[a, b]$ and range a closed interval $[c, d]$. Suppose t is a point in (a, b) such that f has left hand derivative l and right-hand derivative r at t, with both l and r nonzero. What is the left hand derivative and right hand derivative to f^{-1} at $f(t)$? *Last year:* 6/15 *correct*
  (A) The left hand derivative is $1/l$ and the right hand derivative is $1/r$.
  (B) The left hand derivative is $-1/l$ and the right hand derivative is $-1/r$.
  (C) The left hand derivative is $1/r$ and the right hand derivative is $1/l$.
  (D) The left hand derivative is $-1/r$ and the right hand derivative is $-1/l$.
  (E) The left hand derivative is $1/l$ and the right hand derivative is $1/r$ if $l > 0$, otherwise the left hand derivative is $1/r$ and the right hand derivative is $1/l$.

  Your answer: ____________________________

(4) (**) Which of these functions is one-to-one? *Last year:* 2/15 *correct*

(A) $f_1(x) := \{ \begin{array}{rl} x, & x \text{ rational} \\ x^2, & x \text{ irrational} \end{array}$

(B) $f_2(x) := \{ \begin{array}{rl} x, & x \text{ rational} \\ x^3, & x \text{ irrational} \end{array}$

(C) $f_3(x) := \{ \begin{array}{rl} x, & x \text{ rational} \\ 1/(x-1), & x \text{ irrational} \end{array}$

(D) All of the above

(E) None of the above

Your answer: ____________________________

(5) (**) Consider the following function $f : [0,1] \to [0,1]$ given by $f(x) := \{ \begin{array}{rl} \sin(\pi x/2), & 0 \leq x \leq 1/2 \\ \sqrt{x}, & 1/2 < x \leq 1 \end{array}$.

What is the correct expression for $(f^{-1})'(1/2)$? *Last year:* 1/15 *correct*

(A) It does not exist, since the two-sided derivatives of f at 1/2 do not match.

(B) $\sqrt{2}$

(C) $2\sqrt{2}/\pi$

(D) $4/\pi$

(E) $4/(\sqrt{3}\pi)$

Your answer: ____________________________