CLASS QUIZ: NOVEMBER 11: WHOPPERS

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) Suppose $g : \mathbb{R} \to \mathbb{R}$ is a continuous function such that $\lim_{x\to 0} g(x)/x^2 = A$ for some constant $A \neq 0$. What is $\lim_{x\to 0} g(g(x))/x^4$?
   (A) A
   (B) A^2
   (C) A^3
   (D) $A^2 g(A)$
   (E) $g(A)/A^2$
   Your answer: ____________________________

(2) Which of the following statements is **always true**? *Exact replica of a past question.*
   (A) The range of a continuous nonconstant function on a closed bounded interval (i.e., an interval of the form $[a, b]$) is a closed bounded interval (i.e., an interval of the form $[m, M]$).
   (B) The range of a continuous nonconstant function on an open bounded interval (i.e., an interval of the form (a, b)) is an open bounded interval (i.e., an interval of the form (m, M)).
   (C) The range of a continuous nonconstant function on a closed interval that may be bounded or unbounded (i.e., an interval of the form $[a, b]$, $[a, \infty)$, $(-\infty, a]$, or $(-\infty, \infty)$) is also a closed interval that may be bounded or unbounded.
   (D) The range of a continuous nonconstant function on an open interval that may be bounded or unbounded (i.e., an interval of the form (a, b),(a, ∞), $(-\infty, a)$, or $(-\infty, \infty)$), is also an open interval that may be bounded or unbounded.
   (E) None of the above.
   Your answer: ____________________________

(3) Suppose f is a continuously differentiable function on $\mathbb{R}$ and $c \in \mathbb{R}$. Which of the following implications is **false**? *Similar to a past question.*
   (A) If f has mirror symmetry about $x = c$, f' has half turn symmetry about $(c, f'(c))$.
   (B) If f has half turn symmetry about $(c, f(c))$, f' has mirror symmetry about $x = c$.
   (C) If f' has mirror symmetry about $x = c$, f has half turn symmetry about $(c, f(c))$.
   (D) If f' has half turn symmetry about $(c, f'(c))$, f has mirror symmetry about $x = c$.
   (E) None of the above, i.e., they are all true.
   Your answer: ____________________________

(4) Consider the function $f(x) := \{ \begin{matrix} x, & 0 \le x \le 1/2 \\ x - (1/5), & 1/2 < x \le 1 \end{matrix}$. Define by $f^{[n]}$ the function obtained by iterating f n times, i.e., the function $f \circ f \circ f \circ \cdots \circ f$ where f occurs n times. What is the smallest n for which $f^{[n]} = f^{[n+1]}$? *Similar to a question on the previous midterm.*
   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5
   Your answer: ____________________________

(5) With f as in the previous question, what is the set of points in $(0, 1)$ where $f \circ f$ is not continuous?
   (A) 0.5 only
   (B) 0.5 and 0.7

(C) 0.5, 0.7, and 0.9
(D) 0.7 and 0.9
(E) 0.9 only

Your answer: ____________________________

(6) Consider the graph of the function $f(x) := x \sin(1/(x^2 - 1))$. What can we say about the vertical and horizontal asymptotes?
(A) The graph has vertical asymptotes at $x = +1$ and $x = -1$ and horizontal asymptote (in both directions) $y = 0$.
(B) The graph has vertical asymptotes at $x = +1$ and $x = -1$ and horizontal asymptote (in both directions) $y = 1$.
(C) The graph has no vertical asymptotes and horizontal asymptote (in both directions) $y = 0$.
(D) The graph has no vertical asymptotes and horizontal asymptote (in both directions) $y = 1$.
(E) The graph has no vertical or horizontal asymptotes.

Your answer: ____________________________

(7) Suppose f and g are increasing functions from $\mathbb{R}$ to $\mathbb{R}$. Which of the following functions is *not* guaranteed to be an increasing functions from $\mathbb{R}$ to $\mathbb{R}$? *An exact replica of a past question.*
(A) $f + g$
(B) $f \cdot g$
(C) $f \circ g$
(D) All of the above, i.e., none of them is guaranteed to be increasing.
(E) None of the above, i.e., they are all guaranteed to be increasing.

Your answer: ____________________________

(8) Suppose F and G are continuously differentiable functions on all of $\mathbb{R}$ (i.e., both F' and G' are continuous). Which of the following is **not necessarily true**? *Exact replica of a previous question.*
(A) If $F'(x) = G'(x)$ for all integers x, then $F - G$ is a constant function when restricted to integers, i.e., it takes the same value at all integers.
(B) If $F'(x) = G'(x)$ for all numbers x that are not integers, then $F - G$ is a constant function when restricted to the set of numbers x that are not integers.
(C) If $F'(x) = G'(x)$ for all rational numbers x, then $F - G$ is a constant function when restricted to the set of rational numbers.
(D) If $F'(x) = G'(x)$ for all irrational numbers x, then $F - G$ is a constant function when restricted to the set of irrational numbers.
(E) None of the above, i.e., they are all necessarily true.

Your answer: ____________________________

(9) Consider the four functions $\sin(\sin x)$, $\sin(\cos x)$, $\cos(\sin x)$, and $\cos(\cos x)$. Which of the following statements are true about their periodicity?
(A) All four functions are periodic with a period of 2π.
(B) All four functions are periodic with a period of π.
(C) $\sin(\sin x)$ and $\sin(\cos x)$ have a period of π, whereas $\cos(\sin x)$ and $\cos(\cos x)$ have a period of 2π.
(D) $\cos(\sin x)$ and $\cos(\cos x)$ have a period of π, whereas $\sin(\sin x)$ and $\sin(\cos x)$ have a period of 2π.
(E) $\sin(\sin x)$ has a period of 2π, the other three functions have a period of π.

Your answer: ____________________________