CLASS QUIZ: NOVEMBER 4: INTEGRATION

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): _____

(1) Which of the following is an **antiderivative** of $x \cos x$?
(A) $x \sin x + \cos x$
(B) $x \sin x - \cos x$
(C) $-x \sin x + \cos x$
(D) $-x \sin x - \cos x$
(E) None of the above

Your answer: _____

(2) (*) Suppose F and G are two functions defined on $\mathbb{R}$ and k is a natural number such that the k^{th} derivatives of F and G exist and are equal on all of $\mathbb{R}$. Then, $F - G$ must be a polynomial function. What is the **maximum possible degree** of $F - G$? (Note: Assume constant polynomials to have degree zero)
(A) $k - 2$
(B) $k - 1$
(C) k
(D) $k + 1$
(E) There is no bound in terms of k.

Your answer: _____

(3) (**) Suppose f is a continuous function on $\mathbb{R}$. Clearly, f has antiderivatives on $\mathbb{R}$. For all but one of the following conditions, it is possible to guarantee, without any further information about f, that there exists an antiderivative F satisfying that condition. **Identify the exceptional condition** (i.e., the condition that it may not always be possible to satisfy).
(A) $F(1) = F(0)$.
(B) $F(1) + F(0) = 0$.
(C) $F(1) + F(0) = 1$.
(D) $F(1) = 2F(0)$.
(E) $F(1)F(0) = 0$.

Your answer: _____

(4) (**) Suppose $F(x) = \int_0^x \sin^2(t^2)\,dt$ and $G(x) = \int_0^x \cos^2(t^2)\,dt$. Which of the following **is true**?

(A) $F + G$ is the zero function.
(B) $F + G$ is a constant function with nonzero value.
(C) $F(x) + G(x) = x$ for all x.
(D) $F(x) + G(x) = x^2$ for all x.
(E) $F(x^2) + G(x^2) = x$ for all x.

Your answer: _____

(5) (**) Suppose F is a function defined on $\mathbb{R} \setminus \{0\}$ such that $F'(x) = -1/x^2$ for all $x \in \mathbb{R} \setminus \{0\}$. Which of the following pieces of information is/are **sufficient** to determine F completely?

(A) The value of F at any two positive numbers.
(B) The value of F at any two negative numbers.
(C) The value of F at a positive number and a negative number.
(D) Any of the above pieces of information is sufficient, i.e., we need to know the value of F at any two numbers.
(E) None of the above pieces of information is sufficient.

Your answer: _____