CLASS QUIZ: NOVEMBER 2: INTEGRATION

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) Suppose f and g are both functions on $\mathbb{R}$ with the property that f'' and g'' are both everywhere the zero function. For which of the following functions is the second derivative *not necessarily* the zero function everywhere? *Last year:* 14/15 *correct*
(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
(B) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(C) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(D) All of the above, i.e., the second derivative need not be identically zero for any of these functions.
(E) None of the above, i.e., for all these functions, the second derivative is the zero function.

Your answer: ____________________________

(2) Suppose f and g are both functions on $\mathbb{R}$ with the property that f''' and g''' are both everywhere the zero function. For which of the following functions is the third derivative *necessarily* the zero function everywhere? *Last year:* 12/15 *correct*
(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
(B) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(C) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(D) All of the above, i.e., the third derivative is identically zero for all of these functions.
(E) None of the above, i.e., the third derivative is not guaranteed to be the zero function for any of these.

Your answer: ____________________________

(3) Suppose f is a function on an interval $[a, b]$, that is continuous except at finitely many interior points $c_1 < c_2 < \dots < c_n$ ($n \ge 1$), where it has jump discontinuities (hence, both the left-hand limit and the right-hand limit exist but are not equal). Define $F(x) := \int_a^x f(t)\,dt$. Which of the following **is true**? *Last year:* 8/15 *correct*
(A) F is continuously differentiable on (a, b) and the derivative equals f wherever f is continuous.
(B) F is differentiable on (a, b) but the derivative is not continuous, and $F' = f$ on the entire interval.
(C) F has one-sided derivatives on (a, b) and the left-hand derivative of F at any point equals the left-hand limit of f at that point, while the right-hand derivative of F at any point equals the right-hand limit of f at that point.

(D) F has one-sided derivatives on all points of (a, b) except at the points $c_1, c_2, \ldots, c_n$; it is continuous at all these points but does not have one-sided derivatives.
(E) F is continuous at all points of (a, b) except at the points $c_1, c_2, \ldots, c_n$.

Your answer: ____________________________

(4) (**) For a continuous function f on $\mathbb{R}$ and a real number a, define $F_{f,a}(x) = \int_a^x f(t)\,dt$. Which of the following is **true**? *Last year:* 5/15 *correct*

(A) For every continuous function f and every real number a, $F_{f,a}$ is an antiderivative for f, and every antiderivative of f can be obtained in this way by choosing a suitably.
(B) For every continuous function f and every real number a, $F_{f,a}$ is an antiderivative for f, but it is not necessary that every antiderivative of f can be obtained in this way by choosing a suitably. (i.e., there are continuous functions f where not every antiderivative can be obtained in this way).
(C) For every continuous function f, every antiderivative of f can be written as $F_{f,a}$ for some suitable a, but there may be some choices of f and a for which $F_{f,a}$ is not an antiderivative of f.
(D) There may be some choices for f and a for which $F_{f,a}$ is not an antiderivative for f, and there may be some choices of f for which there exist antiderivatives that cannot be written in the form $F_{f,a}$.
(E) None of the above.

Your answer: ____________________________

(5) (**) Suppose F is a differentiable function on an open interval (a, b) and F' is not a continuous function. Which of these discontinuities can F' have? *Last year:* 0/15 *correct*

(A) A removable discontinuity (the limit exists and is finite but is not equal to the value of the function)
(B) An infinite discontinuity (one or both the one-sided limits is infinite)
(C) A jump discontinuity (both one-sided limits exist and are finite, but not equal)
(D) All of the above
(E) None of the above

Your answer: ____________________________