CLASS QUIZ: OCTOBER 28: INTEGRATION BASICS

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

(1) Consider the function(s) $[0,1] \to \mathbb{R}$. **Identify the functions** for which the integral (using upper sums and lower sums) is not defined. *Last year:* 15/15 *correct*
   (A) $f_1(x) := \begin{cases} 0, & 0 \le x < 1/2 \\ 1, & 1/2 \le x \le 1 \end{cases}$.
   (B) $f_2(x) := \begin{cases} 0, & x \ne 0 \text{ and } 1/x \text{ is an integer} \\ 1, & \text{otherwise} \end{cases}$.
   (C) $f_3(x) := \begin{cases} 0, & x \text{ rational} \\ 1, & x \text{ irrational} \end{cases}$
   (D) All of the above
   (E) None of the above

   Your answer: ______________________

(2) (**) Suppose $a < b$. Recall that a *regular partition* into n parts of $[a,b]$ is a partition $a = x_0 < x_1 < \dots < x_{n-1} < x_n = b$ where $x_i - x_{i-1} = (b-a)/n$ for all $1 \le i \le n$. A partition P_1 is said to be a *finer partition* than a partition P_2 if the set of points of P_1 contains the set of points of P_2. Which of the following is a **necessary and sufficient condition** for the regular partition into m parts to be a *finer partition* than the regular partition into n parts? (Note: We'll assume that any partition is finer than itself). *Last year:* 5/15 *correct*
   (A) $m \le n$
   (B) $n \le m$
   (C) m divides n (i.e., n is a multiple of m)
   (D) n divides m (i.e., m is a multiple of n)
   (E) m is a power of n

   Your answer: ______________________

PLEASE TURN OVER FOR THIRD AND FOURTH QUESTIONS

(3) (**) For a partition $P = x_0 < x_1 < x_2 < \cdots < x_n$ of $[a, b]$ (with $x_0 = a$, $x_n = b$) define the norm $\|P\|$ as the maximum of the values $x_i - x_{i-1}$. Which of the following **is always true** for any continuous function f on $[a, b]$? *Last year:* 4/15 *correct*

(A) If P_1 is a finer partition than P_2, then $\|P_2\| \leq \|P_1\|$ (Here, *finer* means that, as a set, $P_2 \subseteq P_1$, i.e., all the points of P_2 are also points of P_1).

(B) If $\|P_2\| \leq \|P_1\|$, then $L_f(P_2) \leq L_f(P_1)$ (where L_f is the lower sum).

(C) If $\|P_2\| \leq \|P_1\|$, then $U_f(P_2) \leq U_f(P_1)$ (where U_f is the upper sum).

(D) If $\|P_2\| \leq \|P_1\|$, then $L_f(P_2) \leq U_f(P_1)$.

(E) All of the above.

Your answer: ____________________________

(4) (**) Suppose F and G are continuously differentiable functions on all of $\mathbb{R}$ (i.e., both F' and G' are continuous). Which of the following is **not necessarily true**? *Last year:* 0/15 *correct*

(A) If $F'(x) = G'(x)$ for all integers x, then $F - G$ is a constant function when restricted to integers, i.e., it takes the same value at all integers.

(B) If $F'(x) = G'(x)$ for all numbers x that are not integers, then $F - G$ is a constant function when restricted to the set of numbers x that are not integers.

(C) If $F'(x) = G'(x)$ for all rational numbers x, then $F - G$ is a constant function when restricted to the set of rational numbers.

(D) If $F'(x) = G'(x)$ for all irrational numbers x, then $F - G$ is a constant function when restricted to the set of irrational numbers.

(E) None of the above, i.e., they are all necessarily true.

Your answer: ____________________________