CLASS QUIZ: OCTOBER 24: CONCAVE, INFLECTIONS, TANGENTS, CUSPS, ASYMPTOTES

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) Consider the function $f(x) := x^3(x-1)^4(x-2)^2$. Which of the following **is true**? *Last year:* 11/15 *correct*
(A) 0, 1, and 2 are all critical points and all of them are points of local extrema.
(B) 0, 1, and 2 are all critical points, but only 0 is a point of local extremum.
(C) 0, 1, and 2 are all critical points, but only 1 and 2 are points of local extrema.
(D) 0, 1, and 2 are all critical points, and none of them is a point of local extremum.
(E) 1 and 2 are the only critical points.

Your answer: ____________________________

(2) Suppose f and g are continuously differentiable functions on $\mathbb{R}$. Suppose f and g are both concave up. Which of the following is **always true**? *Last year:* 8/15 *correct*
(A) $f + g$ is concave up.
(B) $f - g$ is concave up.
(C) $f \cdot g$ is concave up.
(D) $f \circ g$ is concave up.
(E) All of the above.

Your answer: ____________________________

(3) Consider the function $p(x) := x(x-1)\dots(x-n)$, where $n \geq 1$ is a positive integer. How many points of inflection does p have? *Last year:* 7/15 *correct*
(A) $n - 3$
(B) $n - 2$
(C) $n - 1$
(D) n
(E) $n + 1$

Your answer: ____________________________

(4) Suppose f is a polynomial function of degree $n \geq 2$. What can you say about the sense of concavity of the function f for **large enough inputs**, i.e., as $x \to +\infty$? (Note that if $n \leq 1$, f is linear so we do not have concavity in either sense). *Last year:* 12/15 *correct*
(A) f is eventually concave up.
(B) f is eventually concave down.

(C) f is eventually either concave up or concave down, and which of these cases occurs depends on the sign of the leading coefficient of f.
(D) f is eventually either concave up or concave down, and which of these cases occurs depends on whether the degree of f is even or odd.
(E) f may be concave up, concave down, or neither.

Your answer: ____________________________

(5) (**) Suppose f is a continuously differentiable function on $[a, b]$ and f' is continuously differentiable at all points of $[a, b]$ except an interior point c, where it has a vertical cusp. What can we say is **definitely true** about the behavior of f at c? *Last year:* 3/15 *correct*
(A) f attains a local extreme value at c.
(B) f has a point of inflection at c.
(C) f has a critical point at c that does not correspond to a local extreme value.
(D) f has a vertical tangent at c.
(E) f has a vertical cusp at c.

Your answer: ____________________________

(6) (**) Suppose f and g are continuous functions on $\mathbb{R}$, such that f attains a vertical tangent at a and is continuously differentiable everywhere else, and g attains a vertical tangent at b and is continuously differentiable everywhere else. Further, $a \neq b$. What can we say is **definitely true** about $f - g$? *Last year:* 5/15 *correct*
(A) $f - g$ has vertical tangents at a and b.
(B) $f - g$ has a vertical tangent at a and a vertical cusp at b.
(C) $f - g$ has a vertical cusp at a and a vertical tangent at b.
(D) $f - g$ has no vertical tangents and no vertical cusps.
(E) $f - g$ has either a vertical tangent or a vertical cusp at the points a and b, but it is not possible to be more specific without further information.

Your answer: ____________________________

(7) (**) Suppose f and g are continuous functions on $\mathbb{R}$, such that f is continuously differentiable everywhere and g is continuously differentiable everywhere except at c, where it has a vertical tangent. What can we say is **definitely true** about $f \circ g$? *Last year:* 3/15 *correct*
(A) It has a vertical tangent at c.
(B) It has a vertical cusp at c.
(C) It has either a vertical tangent or a vertical cusp at c.
(D) It has neither a vertical tangent nor a vertical cusp at c.
(E) We cannot say anything for certain.

Your answer: ____________________________