CLASS QUIZ: OCTOBER 19: INCREASE/DECREASE AND MAXIMA/MINIMA

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____
You are expected to take about one minute per question.

(1) (*) Suppose f is a function defined on a closed interval $[a, c]$. Suppose that the left-hand derivative of f at c exists and equals ℓ. Which of the following implications is **true in general**? *Last year:* 8/15 *correct*
   (A) If $f(x) < f(c)$ for all $a \le x < c$, then $\ell < 0$.
   (B) If $f(x) \le f(c)$ for all $a \le x < c$, then $\ell \le 0$.
   (C) If $f(x) < f(c)$ for all $a \le x < c$, then $\ell > 0$.
   (D) If $f(x) \le f(c)$ for all $a \le x < c$, then $\ell \ge 0$.
   (E) None of the above is true in general.

   Your answer: ____________________________

(2) (**) Suppose f and g are increasing functions from $\mathbb{R}$ to $\mathbb{R}$. Which of the following functions is *not* guaranteed to be an increasing function from $\mathbb{R}$ to $\mathbb{R}$? *Last year:* 1/15 *correct*
   (A) $f + g$
   (B) $f \cdot g$
   (C) $f \circ g$
   (D) All of the above, i.e., none of them is guaranteed to be increasing.
   (E) None of the above, i.e., they are all guaranteed to be increasing.

   Your answer: ____________________________

PLEASE TURN OVER FOR THE THIRD AND FOURTH QUESTION.

(3) (**) Suppose f is a continuous function defined on an open interval (a, b) and c is a point in (a, b). Which of the following implications is **true**? *Last year:* 5/15 *correct*

(A) If c is a point of local minimum for f, then there is a value $\delta > 0$ and an open interval $(c - \delta, c + \delta) \subseteq (a, b)$ such that f is non-increasing on $(c - \delta, c)$ and non-decreasing on $(c, c + \delta)$.

(B) If there is a value $\delta > 0$ and an open interval $(c - \delta, c + \delta) \subseteq (a, b)$ such that f is non-increasing on $(c - \delta, c)$ and non-decreasing on $(c, c + \delta)$, then c is a point of local minimum for f.

(C) If c is a point of local minimum for f, then there is a value $\delta > 0$ and an open interval $(c - \delta, c + \delta) \subseteq (a, b)$ such that f is non-decreasing on $(c - \delta, c)$ and non-increasing on $(c, c + \delta)$.

(D) If there is a value $\delta > 0$ and an open interval $(c - \delta, c + \delta) \subseteq (a, b)$ such that f is non-decreasing on $(c - \delta, c)$ and non-increasing on $(c, c + \delta)$, then c is a point of local minimum for f.

(E) All of the above are true.

Your answer: ____________________________

(4) (**) Suppose f is a continuously differentiable function on $\mathbb{R}$ and f' is a periodic function with period h. (Recall that periodic derivative implies that the original function is a sum of ...). Suppose S is the set of points of local maximum for f, and T is the set of local maximum values. Which of the following is **true in general** about the sets S and T? *Last year:* 3/15 *correct*

(A) The set S is invariant under translation by h (i.e., $x \in S$ if and only if $x + h \in S$) and all the values in the set T are in the image of the set $[0, h]$ under f.

(B) The set S is invariant under translation by h (i.e., $x \in S$ if and only if $x + h \in S$) but all the values in the set T need not be in the image of the set $[0, h]$ under f.

(C) Both the sets S and T are invariant under translation by h.

(D) Both the sets S and T are finite.

(E) Both the sets S and T are infinite.

Your answer: ____________________________