# CLASS QUIZ: OCTOBER 7: LIMIT THEOREMS

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________________

Questions marked with a (*) are questions that are somewhat trickier, with the probability of getting the question correct being about 50% or less. For these questions, you are free to discuss the questions with others while making your attempt.

Questions marked with a (**) are questions where, in a previous administration of this quiz, a specific incorrect option was chosen by as many people as or more people than the correct option. For these questions, you are free to discuss the questions with others while making your attempt.

(1) (**) Which of the following statements is **always true**? *Last year's performance:* 2/11 *correct*
   (A) The range of a continuous nonconstant function on a closed bounded interval (i.e., an interval of the form $[a, b]$) is a closed bounded interval (i.e., an interval of the form $[m, M]$).
   (B) The range of a continuous nonconstant function on an open bounded interval (i.e., an interval of the form $(a, b)$) is an open bounded interval (i.e., an interval of the form $(m, M)$).
   (C) The range of a continuous nonconstant function on a closed interval that may be bounded or unbounded (i.e., an interval of the form $[a, b]$, $[a, \infty)$, $(-\infty, a]$, or $(-\infty, \infty)$) is also a closed interval that may be bounded or unbounded.
   (D) The range of a continuous nonconstant function on an open interval that may be bounded or unbounded (i.e., an interval of the form $(a, b)$,$(a, \infty)$, $(-\infty, a)$, or $(-\infty, \infty)$), is also an open interval that may be bounded or unbounded.
   (E) None of the above.

   Your answer: ____________________________

(2) (**) Suppose $g : \mathbb{R} \to \mathbb{R}$ is a continuous function such that $\lim_{x \to 0} g(x)/x = A$ for some constant $A \neq 0$. What is $\lim_{x \to 0} g(g(x))/x$? *Last year's performance:* 1/12 *correct*
   (A) $0$
   (B) $A$
   (C) $A^2$
   (D) $g(A)$
   (E) $g(A)/A$

   Your answer: ____________________________

(3) Suppose $I = (a, b)$ is an open interval. A function $f : I \to \mathbb{R}$ is termed *piecewise continuous* if there eixst points $a_0 < a_1 < a_2 < \cdots < a_n$ (dependent on $f$) with $a = a_0$ and $a_n = b$, such that $f$ is continuous on each interval $(a_i, a_{i+1})$. In other words, $f$ is continuous at every point in $(a, b)$ except possibly the $a_i$s.

   Suppose $f$ and $g$ are piecewise continuous functions on the same interval $I$ (with possibly different sets of $a_i$s). Which of the following is/are guaranteed to be piecewise continuous on $I$? *Last year's performance:* 9/11 *correct*
   (A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
   (B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$

(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(D) All of the above
(E) None of the above

Your answer: ____________________________

(4) Suppose $f$ and $g$ are everywhere defined and $\lim_{x\to 0} f(x) = 0$. Which of these pieces of information is **not sufficient** to conclude that $\lim_{x\to 0} f(x)g(x) = 0$? *Last year's performance:* 8/11 *correct*
(A) $\lim_{x\to 0} g(x) = 0$.
(B) $\lim_{x\to 0} g(x)$ is a constant not equal to zero.
(C) There exists $\delta > 0$ and $B > 0$ such that for $0 < |x| < \delta$, $|g(x)| < B$.
(D) $\lim_{x\to 0} g(x) = \infty$, i.e., for every $N > 0$ there exists $\delta > 0$ such that if $0 < |x| < \delta$, then $g(x) > N$.
(E) None of the above, i.e., they are all sufficient to conclude that $\lim_{x\to 0} f(x)g(x) = 0$.

Your answer: ____________________________

(5) $f$ and $g$ are functions defined for all real values. $c$ is a real number. Which of these statements is ***not* necessarily true**? *Last year's performance:* 9/11 *correct*
(A) If $\lim_{x\to c^-} f(x) = L$ and $\lim_{x\to c^-} g(x) = M$, then $\lim_{x\to c^-}(f(x) + g(x))$ exists and is equal to $L + M$.
(B) If $\lim_{x\to c^-} g(x) = L$ and $\lim_{x\to L^-} f(x) = M$, then $\lim_{x\to c^-} f(g(x)) = M$.
(C) If there exists an open interval containing $c$ on which $f$ is continuous and there exists an open interval containing $c$ on which $g$ is continuous, then there exists an open interval containing $c$ on which $f + g$ is continuous.
(D) If there exists an open interval containing $c$ on which $f$ is continuous and there exists an open interval containing $c$ on which $g$ is continuous, then there exists an open interval containing $c$ on which the product $f \cdot g$ (i.e., the function $x \mapsto f(x)g(x)$) is continuous.
(E) None of the above, i.e., they are all necessarily true.

Your answer: ____________________________