CLASS QUIZ: OCTOBER 3: LIMITS

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) Which of these is the correct interpretation of $\lim_{x \to c} f(x) = L$ in terms of the definition of limit? *Last year's performance:* 9/12 *correct*
   (A) For every $\alpha > 0$, there exists $\beta > 0$ such that if $0 < |x - c| < \alpha$, then $|f(x) - L| < \beta$.
   (B) There exists $\alpha > 0$ such that for every $\beta > 0$, and $0 < |x - c| < \alpha$, we have $|f(x) - L| < \beta$.
   (C) For every $\alpha > 0$, there exists $\beta > 0$ such that if $0 < |x - c| < \beta$, then $|f(x) - L| < \alpha$.
   (D) There exists $\alpha > 0$ such that for every $\beta > 0$ and $0 < |x - c| < \beta$, we have $|f(x) - L| < \alpha$.
   (E) None of the above

   Your answer: ____________________________

(2) Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function. Which of the following says that f does not have a limit at any point in $\mathbb{R}$ (i.e., there is no point $c \in \mathbb{R}$ for which $\lim_{x \to c} f(x)$ exists)? *Last year's performance:* 10/12 *correct*
   (A) For every $c \in \mathbb{R}$, there exists $L \in \mathbb{R}$ such that for every $\epsilon > 0$, there exists $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, we have $|f(x) - L| \geq \epsilon$.
   (B) There exists $c \in \mathbb{R}$ such that for every $L \in \mathbb{R}$, there exists $\epsilon > 0$ such that for every $\delta > 0$, there exists x satisfying $0 < |x - c| < \delta$ and $|f(x) - L| \geq \epsilon$.
   (C) For every $c \in \mathbb{R}$ and every $L \in \mathbb{R}$, there exists $\epsilon > 0$ such that for every $\delta > 0$, there exists x satisfying $0 < |x - c| < \delta$ and $|f(x) - L| \geq \epsilon$.
   (D) There exists $c \in \mathbb{R}$ and $L \in \mathbb{R}$ such that for every $\epsilon > 0$, there exists $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, we have $|f(x) - L| \geq \epsilon$.
   (E) All of the above.

   Your answer: ____________________________

(3) In the usual $\epsilon - \delta$ definition of limit for a given limit $\lim_{x \to c} f(x) = L$, if a given value $\delta > 0$ works for a given value $\epsilon > 0$, then which of the following is true? *Last year's performance:* 17/26 *correct, appeared in 153 quiz*
   (A) Every smaller positive value of δ works for the same ϵ. Also, the given value of δ works for every smaller positive value of ϵ.
   (B) Every smaller positive value of δ works for the same ϵ. Also, the given value of δ works for every larger value of ϵ.
   (C) Every larger value of δ works for the same ϵ. Also, the given value of δ works for every smaller positive value of ϵ.
   (D) Every larger value of δ works for the same ϵ. Also, the given value of δ works for every larger value of ϵ.
   (E) None of the above statements need always be true.

   Your answer: ____________________________

(4) Which of the following is a correct formulation of the statement $\lim_{x\to c} f(x) = L$, in a manner that avoids the use of ϵs and δs? *Not appeared in previous years*

(A) For every open interval centered at c, there is an open interval centered at L such that the image under f of the open interval centered at c (excluding the point c itself) is contained in the open interval centered at L.

(B) For every open interval centered at c, there is an open interval centered at L such that the image under f of the open interval centered at c (excluding the point c itself) contains the open interval centered at L.

(C) For every open interval centered at L, there is an open interval centered at c such that the image under f of the open interval centered at c (excluding the point c itself) is contained in the open interval centered at L.

(D) For every open interval centered at L, there is an open interval centered at c such that the image under f of the open interval centered at c (excluding the point c itself) contains the open interval centered at L.

(E) None of the above.

Your answer: ____________________________