

CLASS QUIZ: SEPTEMBER 30: LIMITS

VIPUL NAIK

Your name (print clearly in capital letters): _____

Write your answer in the space provided. In the space below, you can explain your work if you want (this will not affect scoring). I may or may not get time to look at the work you have done, but it may help you recall how you arrived at a particular answer.

You are expected to take about one minute per question.

Questions marked with a (*) are questions that are somewhat trickier, with the probability of getting the question correct being about 50% or less. For these questions, you are free to discuss the questions with others while making your attempt.

Questions marked with a (**) are questions where, in a previous administration of this quiz, a specific incorrect option was chosen by as many people as or more people than the correct option. For these questions, you are free to discuss the questions with others while making your attempt.

- (1) (**) We call a function f left continuous on an open interval I if, for all $a \in I$, $\lim_{x \rightarrow a^-} f(x) = f(a)$. Which of the following is an example of a function that is left continuous but not continuous on $(0, 1)$? *Last year's performance: 6/13 correct*
- (A) $f(x) := \begin{cases} x, & 0 < x \leq 1/2 \\ 2x, & 1/2 < x < 1 \end{cases}$
- (B) $f(x) := \begin{cases} x, & 0 < x < 1/2 \\ 2x, & 1/2 \leq x < 1 \end{cases}$
- (C) $f(x) := \begin{cases} x, & 0 < x \leq 1/2 \\ 2x - (1/2), & 1/2 < x < 1 \end{cases}$
- (D) $f(x) := \begin{cases} x, & 0 < x < 1/2 \\ 2x - (1/2), & 1/2 \leq x < 1 \end{cases}$
- (E) All of the above

Your answer: _____

- (2) (**) Suppose f and g are functions $(0, 1)$ to $(0, 1)$ that are both left continuous on $(0, 1)$. Which of the following is *not* guaranteed to be left continuous on $(0, 1)$? *Last year's performance: 4/13 correct*
- (A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
- (B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$
- (C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
- (D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
- (E) None of the above, i.e., they are all guaranteed to be left continuous functions

Your answer: _____

(3) (*) Consider the function:

$$f(x) := \begin{cases} x, & x \text{ rational} \\ 1/x, & x \text{ irrational} \end{cases}$$

What is the set of all points at which f is continuous? *Last year's performance: 5/13 correct*

- (A) $\{0, 1\}$
- (B) $\{-1, 1\}$
- (C) $\{-1, 0\}$
- (D) $\{-1, 0, 1\}$
- (E) f is continuous everywhere

Your answer: _____