CLASS QUIZ: SEPTEMBER 28; TOPIC: FUNCTIONS

VIPUL NAIK

Your name (print clearly in capital letters): ____

Write your answer in the space provided. In the space below, you can explain your work if you want (this will not affect scoring). I may or may not get time to look at the work you have done, but it may help you recall how you arrived at a particular answer.

You are expected to take about one minute per question.

Questions marked with a (*) are questions that are somewhat trickier, with the probability of getting the question correct being about 50% or less. For these questions, you are free to discuss the questions with others while making your attempt.

(1) Suppose f and g are functions from $\mathbb{R}$ to $\mathbb{R}$. Suppose both f and g are even, i.e., $f(x) = f(-x)$ for all $x \in \mathbb{R}$ and $g(x) = g(-x)$ for all $x \in \mathbb{R}$. Which of the following is *not* guaranteed to be an even function from the given information? *Last year's performance*: 11/15 correct

Note: For this question, it is possible to solve the question by taking a few simple examples. You're free to do this, but the recommended method for tackling the question is to handle it **abstractly***, i.e., try to prove or disprove in general for each function whether it is even.*

(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
(B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$
(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(E) None of the above, i.e., they are all guaranteed to be even functions.

Your answer: ____________________________

(2) (*) Suppose f and g are functions from $\mathbb{R}$ to $\mathbb{R}$. Suppose both f and g are odd, i.e., $f(-x) = -f(x)$ for all $x \in \mathbb{R}$ and $g(-x) = -g(x)$ for all $x \in \mathbb{R}$. Which of the following is *not* guaranteed to be an odd function from the given information? *Last year's performance*: 7/15 correct

Note: For this question, it is possible to solve the question by taking a few simple examples. You're free to do this, but the recommended method for tackling the question is to handle it **abstractly***, i.e., try to prove or disprove in general for each function whether it is odd.*

(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
(B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$
(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(E) None of the above, i.e., they are all guaranteed to be odd functions.

Your answer: ____________________________

(3) For which of the following pairs of polynomial functions f and g is it true that $f \circ g \neq g \circ f$? *Last year's performance*: 14/15 correct
(A) $f(x) := x^2$ and $g(x) := x^3$
(B) $f(x) := x + 1$ and $g(x) := x + 2$
(C) $f(x) := x^2 + 1$ and $g(x) := x^2 + 1$
(D) $f(x) := -x$ and $g(x) := x^2$
(E) $f(x) := -x$ and $g(x) := x^3$

Your answer: ____________________________

(4) (*) Which of the following functions is *not* periodic? *Last year's performance*: 8/15 correct
(A) $\sin(x^2)$
(B) $\sin^2 x$
(C) $\sin(\sin x)$
(D) $\sin(x + 13)$
(E) $(\sin x) + 13$

Your answer: ____________________________

(5) (*) What is the domain of the function $\sqrt{1-x} + \sqrt{x-2}$? Here, domain refers to the *largest subset of the reals* on which the function can be defined. *Last year's performance*: 8/15 correct

Hint: Think clearly, first about what the domain of each of the two functions being added is, and then about whether you need to take the union or the intersection of the domains of the individual functions.
(A) $(1, 2)$
(B) $[1, 2]$
(C) $(-\infty, 1) \cup (2, \infty)$
(D) $(-\infty, 1] \cup [2, \infty)$
(E) None of the above

Your answer: ____________________________