TAKE-HOME CLASS QUIZ: DUE MONDAY OCTOBER 14: MATRIX COMPUTATIONS

MATH 196, SECTION 57 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

PLEASE DO *NOT* DISCUSS ANY QUESTIONS EXCEPT THE STARRED OR DOUBLE-STARRED QUESTIONS.

This quiz has a few questions on the mechanics of the computational execution of Gauss-Jordan elimination, and it has one question on setting up a linear system.

Suppose f is a function on the positive integers that takes positive integer values. Suppose n is a parameter related to the input size of an algorithm. We say that the running time of an algorithm (respectively, the space requirement of the algorithm) is:

- $O(f(n))$ if, for large enough n, it can be bounded from above by a positive constant times $f(n)$.
- $\Omega(f(n))$ if, for large enough n, it can be bounded from below by a positive constant times $f(n)$.
- $\Theta(f(n))$ if it is both $O(f(n))$ and $\Omega(f(n))$.

You can read more at:
`http://en.wikipedia.org/wiki/Big_O_notation`

(1) (*) If you treat each arithmetic operation (addition, subtraction, multiplication, division) of numbers as taking constant time, and all entry rewrites and changes as again taking constant time per entry, what would be the best description of the worst-case running time of the algorithm to convert a $n \times n$ matrix to reduced row-echelon form? (Note that this complexity is termed *arithmetic complexity* and can be distinguished from the *bit complexity* of the algorithm, which could be considerably higher).
(A) $\Theta(n)$
(B) $\Theta(n^2)$
(C) $\Theta(n^3)$
(D) $\Theta(n^4)$
(E) $\Theta(n^5)$

Your answer: ______________________

(2) (*) If you treat each arithmetic operation (addition, subtraction, multiplication, division) of numbers as taking constant space, and all matrix entries as taking constant space, what would be the best description of the worst-case space requirement of the algorithm to convert a $n \times n$ matrix to reduced row-echelon form? Assume that space is reusable, i.e., it is possible to rewrite over existing space used.
(A) $\Theta(n)$
(B) $\Theta(n^2)$
(C) $\Theta(n^3)$
(D) $\Theta(n^4)$
(E) $\Theta(n^5)$

Your answer: ______________________

(3) (*) Suppose the coefficient matrix of a linear system with n variables and n equations is known in advance, and we can spend as much time processing it as we desire in advance (this time will not count towards the running time of the algorithm). In other words, we can use Gauss-Jordan elimination to row-reduce the coefficient matrix in advance. However, we do not have the output column with us in advance. What is the worst-case running time of the part of the algorithm that runs after the output column is known?

(A) $\Theta(n)$
(B) $\Theta(n^2)$
(C) $\Theta(n^3)$
(D) $\Theta(n^4)$
(E) $\Theta(n^5)$

Your answer: ____________________________

(4) *Do not discuss this!*: Which of the following matrices does *not* have the identity matrix as its reduced row-echelon form?

(A)

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

(B)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 7 \end{bmatrix}$$

(C)

$$\begin{bmatrix} 4 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 5 & -6 \end{bmatrix}$$

(D)

$$\begin{bmatrix} 1 & 2 & -3 \\ 4 & -3 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

(E)

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 7 & 11 \end{bmatrix}$$

Your answer: ____________________________

(5) *Do not discuss this!*: A number of different consumer price indices have been constructed. All of them use the market prices for an existing collection of commodities (though not all of them use every commodity in the collection) and take a different "weighted" linear combination of those. For instance, one price index might be 3 times (the price per ton of wheat on the Chicago wheat market) + 4 times (the price of 1 gallon of unleaded gasoline at a particular gas station) + 17 times (the price of Burt's chapstick). Another price index might use 30 times (the price of Transcend's 32 GB flash drive) + 14 times (the price of 1 gallon of gasoline at a particular gas station).

What is a good way of modeling these?

(A) The prices of the various goods are stored in a matrix, the different weightings used in various indices are stored in a vector, and the consumer price indices arise as the output vector of the matrix-vector product.
(B) The different weightings used in various indices are stored in a matrix, the prices of the various goods are stored in a vector, and the consumer price indices arise as the output vector of the matrix-vector product.
(C) The prices of the various goods are stored in a matrix, the consumer price indices are stored as a vector, and the weightings used in the indices arise as the output vector of the matrix-vector product.
(D) The different weightings used in various indices are stored in a matrix, the consumer price indices are stored in a vector, and the prices of the various goods arise as the output vector of the matrix-vector product.

(E) The consumer price indices are stored in a matrix, the prices of the various goods are stored in a vector, and the weightings used in the indices arise as the output vector of the matrix-vector product.

Your answer: ____________________________