TAKE-HOME CLASS QUIZ: DUE MONDAY FEBRUARY 18: PARTIAL DERIVATIVES

MATH 195, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

YOU ARE FREE TO DISCUSS ALL QUESTIONS, BUT PLEASE ONLY ENTER FINAL ANSWER OPTIONS THAT YOU PERSONALLY ENDORSE. DO NOT ENGAGE IN GROUPTHINK.

(1) For this and the next question, consider the function on $\mathbb{R}^2$ given as:

$$f(x,y) := \begin{cases} 1, & x \text{ rational or } y \text{ rational} \\ 0, & x \text{ and } y \text{ both irrational} \end{cases}$$

What can we say about the subset S of $\mathbb{R}^2$ defined as the set of points where f_x is defined?

(A) S is the set of points for which at least one coordinate is rational.
(B) S is the set of points for which both coordinates are rational.
(C) S is the set of points for which the x-coordinate is rational.
(D) S is the set of points for which the y-coordinate is rational.
(E) S is the set of points for which at least one coordinate is irrational.

Your answer: ______________________

(2) With f as in the previous question, what is the subset T of $\mathbb{R}^2$ at which the second-order mixed partial derivative f_{xy} is defined?

(A) T is the empty subset.
(B) T is the set of points for which both coordinates are rational.
(C) T is the set of points for which the x-coordinate is rational.
(D) T is the set of points for which the y-coordinate is rational.
(E) T is the set of points for which both coordinates are irrational.

Your answer: ______________________

(3) For this and the next three questions, consider the function on $\mathbb{R}^2$ given as:

$$g(x,y) := \begin{cases} 1, & x \text{ rational} \\ 0, & x \text{ irrational} \end{cases}$$

What can we say about the subset U of $\mathbb{R}^2$ defined as the set of points where g_x is defined?

(A) U is the empty subset.
(B) U is the set of points for which both coordinates are rational.
(C) U is the set of points for which the x-coordinate is rational.
(D) U is the set of points for which the y-coordinate is rational.
(E) U is the whole plane $\mathbb{R}^2$.

Your answer: ______________________

(4) With g as in the preceding question, what can we say about the subset V of $\mathbb{R}^2$ defined as the set of points where g_y is defined?

(A) V is the empty subset.
(B) V is the set of points for which both coordinates are rational.
(C) V is the set of points for which the x-coordinate is rational.
(D) V is the set of points for which the y-coordinate is rational.
(E) V is the whole plane $\mathbb{R}^2$.

Your answer: ____________________________

(5) With g as in the preceding question, what can we say about the subset W of $\mathbb{R}^2$ defined as the set of points where g_{xy} is defined?
(A) W is the empty subset.
(B) W is the set of points for which both coordinates are rational.
(C) W is the set of points for which the x-coordinate is rational.
(D) W is the set of points for which the y-coordinate is rational.
(E) W is the whole plane $\mathbb{R}^2$.

Your answer: ____________________________

(6) With g as in the preceding question, what can we say about the subset X of $\mathbb{R}^2$ defined as the set of points where g_{yx} is defined?
(A) X is the empty subset.
(B) X is the set of points for which both coordinates are rational.
(C) X is the set of points for which the x-coordinate is rational.
(D) X is the set of points for which the y-coordinate is rational.
(E) X is the whole plane $\mathbb{R}^2$.

Your answer: ____________________________

(7) For this and the next two questions, consider the function on $\mathbb{R}^2$ given as:

$$h(x,y) := \begin{cases} 1, & x \text{ an integer or } y \text{ an integer} \\ 0, & x \text{ not an integer and } y \text{ not an integer} \end{cases}$$

What can we say about the subset A of $\mathbb{R}^2$ defined as the set of points where h_{xy} is defined?
(A) A is the empty set.
(B) A is the set of points whose x-coordinate is an integer.
(C) A is the set of points whose x-coordinate is not an integer.
(D) A is the set of points whose y-coordinate is an integer.
(E) A is the set of points whose y-coordinate is not an integer.

Your answer: ____________________________

(8) With h as defined in the previous question, what can we say about the subset B of $\mathbb{R}^2$ defined as the set of points where h_x is defined but h_{xy} is not defined?
(A) B is the empty set.
(B) B is the set of points for which both coordinates are integers.
(C) B is the set of points for which both coordinates are non-integers.
(D) B is the set of points for which at least one coordinate is an integer.
(E) B is the set of points for which at least one coordinate is a non-integer.

Your answer: ____________________________

(9) With h as defined in the previous question, what can we say about the subset C of $\mathbb{R}^2$ defined as the set of points where both h_{xy} and h_{yx} are defined?
(A) C is the empty set.
(B) C is the set of points for which both coordinates are integers.
(C) C is the set of points for which both coordinates are non-integers.
(D) C is the set of points for which at least one coordinate is an integer.
(E) C is the set of points for which at least one coordinate is a non-integer.

Your answer: ____________________________

(10) Students training for an examination can spend money either on purchasing textbooks or on private tuitions. A student's expected performance on the examination is a function of the money the student spends on textbooks and on tuition (viewed as separate variables). Two researchers want to consider the question of whether increased expenditure on textbooks leads to improved performance on the examination, and if so, by how much.

One researcher decides to measure the increase in the examination score for a marginal increase in textbook expenditure *holding constant the expenditure on tuitions*, arguing that in order to determine the effect of changes in textbook expenditures, the other expenditures need to be kept constant.

The other researcher believes that since the student has a limited budget, it would be more realistic to measure the increase in the examination score for a marginal increase in textbook expenditure *holding constant the total expenditure on both textbook and tuitions.* This is because the student is likely to allocate money away from tuition expenditures in order to spend money on textbooks.

Which of the following best describes what's happening?

(A) Both researchers are effectively computing the same quantity.
(B) The two quantities that the researchers are computing have a simple linear relationship, i.e., their sum or difference is a constant.
(C) The two quantities that the researchers are computing are meaningfully different and there is a relationship between them but that relationship involves other partial derivatives.

Your answer: ____________________________

(11) F is an everywhere twice differentiable function of two variables x and y. Which of the following captures the manner in which the inputs x and y *interact* with each other in the description of F?

(A) The difference $F_x - F_y$
(B) The quotient F_x/F_y.
(C) The product F_xF_y.
(D) The product $F_{xx}F_{yy}$.
(E) The mixed partial F_{xy}

Your answer: ____________________________

(12) F is a function of two variables x and y such that both F_x and F_y exist. Which of the following is generically true?

(A) In general, F_x depends only on x (i.e., it is independent of y) and F_y depends only on y. An exception is if F is multiplicatively separable.
(B) In general, F_x depends only on y (i.e., it is independent of x) and F_y depends only on x (i.e., it is independent of y). An exception is if F is multiplicatively separable.
(C) In general, both F_x and F_y could each depend on both x and y. An exception is if F is additively separable, in which case F_x depends only on y and F_y depends only on x.
(D) In general, both F_x and F_y could each depend on both x and y. An exception is if F is additively separable, in which case F_x depends only on x and F_y depends only on y.
(E) In general, either both F_x and F_y depend only on x or both F_x and F_y depend only on y.

Your answer: ____________________________

(13) Consider a production function $f(L, K, T)$ of three inputs L (labor expenditure), K (capital expenditure), and T (technology expenditure). Suppose all mixed partials of f with respect to L, K, and T are continuous. Suppose we have the following signs of partial derivatives: $\partial f/\partial L > 0$, $\partial f/\partial K > 0$, $\partial^2 f/(\partial L \partial K) < 0$, and $\partial^3 f/(\partial L \partial K \partial T) > 0$. What does this mean?

(A) Increasing labor increases production, increasing capital increases production, and labor and capital substitute for each other to some extent. Increasing the expenditure on technology increases the degree to which labor and capital substitute for each other.
(B) Increasing labor increases production, increasing capital increases production, and labor and capital substitute for each other to some extent. Increasing the expenditure on technology decreases the degree to which labor and capital substitute for each other, i.e., with more technology investment, labor and capital become more complementary.
(C) Increasing labor increases production, increasing capital increases production, and labor and capital complement each other to some extent. Increasing the expenditure on technology increases the degree to which labor and capital complement for each other.
(D) Increasing labor increases production, increasing capital increases production, and labor and capital complement each other to some extent. Increasing the expenditure on technology decreases the degree to which labor and capital complement for each other.

(E) Increasing labor or capital decreases production.

Your answer: ____________________________

(14) Analysis of usage of an online social network finds that the total time spent by people on the social network is $P^{1.3}L^{0.5}$ where P is the total number of people on the network and L is a number of processors used at the social network's server facility. Which of these is true?

(A) Increasing returns both on persons and on processors: every new person joining the network increases the average time spent *per person* (and not just the total time), and every new processor added to the server facility increases the average time spent per processor.
(B) Constant returns on persons, increasing returns on processors
(C) Constant returns on persons, decreasing returns on processors
(D) Increasing returns on persons, decreasing returns on processors
(E) Decreasing returns on persons, increasing returns on processors

Your answer: ____________________________

(15) *Not a calculus question, but has deep calculus interpretations – it is basically measuring the derivative of the* $1/x$ *function with respect to* x: A person travels fifty miles every day by car and the travel distance is fixed. The price of gasoline, which she uses to fuel her car, is also fixed. Which of the following increases in fuel efficiency result in the maximum amount of savings for her?

(A) From 11 to 12 miles per gallon
(B) From 12 to 14 miles per gallon
(C) From 20 to 25 miles per gallon
(D) From 36 to 54 miles per gallon
(E) From 50 to 100 miles per gallon

Your answer: ____________________________

(16) For which of the following production functions $f(L, K)$ of labor and capital is it true that labor and capital can be complementary for some choices of (L, K), and substitutes for others? In other words, for which of these are labor and capital neither globally complements nor globally substitutes? Assume the domain $L > 0, K > 0$.

(A) $L^2 + LK + K^2$
(B) $L^2 - LK + K^2$
(C) $L^3 + L^2K + LK^2 + K^3$
(D) $L^3 + L^2K - LK^2 + K^3$
(E) $L^3 - L^2K - LK^2 + K^3$

Your answer: ____________________________

(17) Consider the following Leontief-like production function $f(L, K) = (\min\{L, K\})^2$. Assume the domain $L > 0$, $K > 0$. What is the nature of returns and complementarity here?

(A) Positive increasing returns on the smaller of the inputs, positive constant returns on the larger of the inputs
(B) Positive constant returns of the smaller of the inputs, positive increasing returns on the larger of the inputs
(C) Zero returns on the smaller of the inputs, positive constant returns on the larger of the inputs
(D) Positive decreasing returns on the smaller of the inputs, zero returns on the larger of the inputs
(E) Positive increasing returns on the smaller of the inputs, zero returns on the larger of the inputs

Your answer: ____________________________