TAKE-HOME CLASS QUIZ: DUE WEDNESDAY JANUARY 9: PARAMETRIC STUFF

MATH 195, SECTION 59 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

THIS IS A TAKE-HOME CLASS QUIZ, BUT I WILL GIVE YOU ABOUT 5 MINUTES TO REVIEW YOUR ANSWERS IN CLASS AND DISCUSS WITH OTHER STUDENTS.

YOU ARE ALLOWED TO DISCUSS ONLY QUESTIONS THAT BEGIN WITH A (*) OR (**). PLEASE ATTEMPT ALL OTHER QUESTIONS BY YOURSELF. EVEN FOR THE QUESTIONS YOU DISCUSS, PLEASE FINALLY ENTER ONLY THE ANSWER OPTION YOU ARE PERSONALLY MOST CONVINCED ABOUT – DON'T ENGAGE IN GROUPTHINK.

(1) Consider the curve given by the parametric description $x = \cos t$, $y = \sin t$, where t varies over the interval $[a, b]$ with $a < b$. What is a necessary and sufficient condition on a and b for this curve to be the circle $x^2 + y^2 = 1$? *Last time:* 11/24 *correct*
(A) $b - a = \pi$
(B) $b - a > \pi$
(C) $b - a = 2\pi$
(D) $b - a > 2\pi$
(E) $b - a \geq 2\pi$

Your answer: ____________________________

(2) (**) Consider the curve given by the parametric description $x = \arctan t$ and $y = \arctan t$ for $t \in \mathbb{R}$. Which of the following is the best description of this curve? *Last time:* 8/24 *correct*
(A) It is the graph of the function arctan
(B) It is the line $y = x$
(C) It is a line segment (without endpoints) that is part of the line $y = x$
(D) It is a half-line (with endpoint) that is part of the line $y = x$
(E) It is a disjoint union of two half-lines that are both part of the line $y = x$

Your answer: ____________________________

(3) (**) Consider the curve given by the parametric description $x = \sin^2 t$ and $y = \cos^2 t$ for $t \in \mathbb{R}$. Which of the following is the best description of this curve? *Last time:* 5/24 *correct*
(A) It is the arc of the circle $x^2 + y^2 = 1$ comprising the first quadrant, i.e., when $x \geq 0$ and $y \geq 0$.
(B) It is the entire circle $x^2 + y^2 = 1$
(C) It is the line segment joining the points $(0, 1)$ and $(1, 0)$
(D) It is the line $y = 1 - x$
(E) It is a portion of the parabola $y = x^2$

Your answer: ____________________________

(4) Identify the parametric description which *does not* correspond to the set of points (x, y) satisfying $x^3 = y^5$. *Last time:* 16/24 *correct*
(A) $x = t^3$, $y = t^5$, for $t \in \mathbb{R}$
(B) $x = t^5$, $y = t^3$, for $t \in \mathbb{R}$
(C) $x = t$, $y = t^{3/5}$, for $t \in \mathbb{R}$
(D) $x = t^{5/3}$, $y = t$, for $t \in \mathbb{R}$
(E) All of the above parametric descriptions work

Your answer: ______________________________

(5) (**) Consider the parametric description $x = f(t)$, $y = g(t)$ where t varies over all of $\mathbb{R}$. What is the necessary and sufficient condition for the curve given by this to be the graph of a function, i.e., to satisfy the vertical line test? *Last time:* 10/24 *correct*
(A) For any t_1 and t_2 satisfying $f(t_1) = f(t_2)$, we must have $g(t_1) = g(t_2)$.
(B) For any t_1 and t_2 satisfying $g(t_1) = g(t_2)$, we must have $f(t_1) = f(t_2)$.
(C) Both f and g are one-to-one functions.
(D) For any t_1 and t_2, we must have $f(t_1) = f(t_2)$.
(E) For any t_1 and t_2, we must have $g(t_1) = g(t_2)$.

Your answer: ______________________________

(6) Suppose f and g are both twice differentiable functions everywhere on $\mathbb{R}$. Which of the following is the correct formula for $(f \circ g)''$? *Last time:* 20/21 *correct*
(A) $(f'' \circ g) \cdot g''$
(B) $(f'' \circ g) \cdot (f' \circ g') \cdot g''$
(C) $(f'' \circ g) \cdot (f' \circ g') \cdot (f \circ g'')$
(D) $(f'' \circ g) \cdot (g')^2 + (f' \circ g) \cdot g''$
(E) $(f' \circ g') \cdot (f \circ g) + (f'' \circ g'')$

Your answer: ______________________________

(7) Suppose $x = f(t)$ and $y = g(t)$ where f and g are both twice differentiable functions. What is d^2y/dx^2 in terms of f and g and their derivatives evaluated at t? *Last time:* 20/21 *correct*
(A) $(f'(t)g''(t) - g'(t)f''(t))/(f'(t))^3$
(B) $(f'(t)g''(t) - g'(t)f''(t))/(g'(t))^3$
(C) $(g'(t)f''(t) - f'(t)g''(t))/(f'(t))^3$
(D) $(g'(t)f''(t) - f'(t)g''(t))/(g'(t))^3$
(E) None of the above

Your answer: ______________________________

(8) Which of the following pair of bounds works for the arc length for the portion of the graph of the sine function between $(a, \sin a)$ and $(b, \sin b)$ where $a < b$? *Last time:* 15/21 *correct*
(A) Between $(b - a)/\sqrt{3}$ and $(b - a)/\sqrt{2}$

(B) Between $(b-a)/\sqrt{2}$ and $b-a$
(C) Between $(b-a)$ and $\sqrt{2}(b-a)$
(D) Between $\sqrt{2}(b-a)$ and $\sqrt{3}(b-a)$
(E) Between $\sqrt{3}(b-a)$ and $2(b-a)$

Your answer: ____________________________

(9) (*) Consider the parametric curve $x = e^t$, $y = e^{t^2}$. How does y grow in terms of x as $x \to \infty$? *Last time:* 7/21 *correct*
(A) y grows like a polynomial in x.
(B) y grows faster than any polynomial in x but slower than an exponential function of x.
(C) y grows exponentially in x.
(D) y grows super-exponentially in x but slower than a double exponential in x.
(E) y grows like a double exponential in x.

Your answer: ____________________________

(10) We say that a curve is *algebraic* if it admits a parameterization of the form $x = f(t)$, $y = g(t)$, where f and g are rational functions and t varies over some subset of the real numbers. Which of the following curves is *not* algebraic? *Last time:* 11/21 *correct*
(A) $x = \cos t$, $y = \sin t$, $t \in \mathbb{R}$
(B) $x = \cos t$, $y = \cos(3t)$, $t \in \mathbb{R}$
(C) $x = \cos t$, $y = \cos^2 t$, $t \in \mathbb{R}$
(D) $x = \cos t$, $y = \cos(t^2)$, $t \in \mathbb{R}$
(E) None of the above, i.e., they are all algebraic

Your answer: ____________________________

(11) (**) Suppose $x = f(t)$, $y = g(t)$, $t \in \mathbb{R}$ is a parametric description of a curve Γ and both f and g are continuous on all of $\mathbb{R}$. If both f and g are even, what can we conclude about Γ and its parameterization? *Last time:* 5/21 *correct*
(A) Γ is symmetric about the y-axis
(B) Γ is symmetric about the x-axis
(C) Γ is symmetric about the line $y = x$
(D) Γ has half turn symmetry about the origin
(E) The parameterizations of Γ for $t \leq 0$ and for $t \geq 0$ both cover all of Γ, and in directions mutually reverse to each other.

Your answer: ____________________________