CLASS QUIZ: FEBRUARY 29: SERIES

MATH 153, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) Suppose p is a polynomial that take positive values on all nonnegative integers. Consider the summation $\sum_{k=1}^{\infty} \frac{(k^2+1)^{2/3}}{p(k)}$. Under what conditions does the summation converge? Note that the degree of p must be a nonnegative integer.
(A) The summation converges if and only if the degree of p is *at least* one
(B) The summation converges if and only if the degree of p is *at least* two
(C) The summation converges if and only if the degree of p is *at least* three
(D) The summation converges if and only if the degree of p is *at most* two
(E) The summation converges if and only if the degree of p is *at most* one

Your answer: ____________________________

(2) Suppose p is a polynomial that take positive values on all nonnegative integers. Consider the summation $\sum_{k=1}^{\infty} \frac{(-1)^k(k^2+1)^{2/3}}{p(k)}$. Under what conditions does the summation converge? Note that the degree of p must be a nonnegative integer.
(A) The summation converges if and only if the degree of p is *at least* one
(B) The summation converges if and only if the degree of p is *at least* two
(C) The summation converges if and only if the degree of p is *at least* three
(D) The summation converges if and only if the degree of p is *at most* two
(E) The summation converges if and only if the degree of p is *at most* one

Your answer: ____________________________

(3) (*) Which of the following series converges? Assume for all series that the starting point of summation is large enough that the terms are well defined. *Last year:* 11/25 *correct*
(A) $\sum 1/(k\ln(\ln k))$
(B) $\sum 1/(k\ln k)$
(C) $\sum 1/(k(\ln(\ln k))^2)$
(D) $\sum 1/(k(\ln k)(\ln(\ln k)))$
(E) $\sum 1/(k(\ln k)(\ln(\ln k))^2)$

Your answer: ____________________________

(4) Which of the following series converges? *Last year:* 23/25 *correct*
(A) $\sum \frac{k+\sin k}{k^2+1}$
(B) $\sum \frac{k+\cos k}{k^3+1}$
(C) $\sum \frac{k^2-\sin k}{k+1}$
(D) $\sum \frac{k^3+\cos k}{k^2+1}$
(E) $\sum \frac{k}{\sin(k^3+1)}$

Your answer: ____________________________

(5) Consider the series $\sum_{k=0}^{\infty} \frac{1}{2^{2^k}}$. What can we say about the sum of this series? *Last year:* 14/26 *correct*
(A) It is finite and strictly between 0 and 1.

(B) It is finite and equal to 1.
(C) It is finite and strictly between 1 and 2.
(D) It is finite and equal to 2.
(E) It is infinite.

Your answer: ____________________________

(6) For one of the following functions f on $(0, \infty)$, the integral $\int_0^\infty f(x)\,dx$ converges but $\int_0^\infty |f(x)|\,dx$ does not converge. What is that function f? (Note that this is similar to, but not quite the same as, the absolute versus conditional convergence notion for series).
(A) $f(x) = \sin x$
(B) $f(x) = \sin(\sin x)$
(C) $f(x) = (\sin \sqrt{x})/\sqrt{x}$
(D) $f(x) = (\sin x)/x$
(E) $f(x) = (\sin^3 x)/x^3$

Your answer: ____________________________

(7) (**) Consider the function $F(x, p) := \sum_{n=1}^\infty \frac{x^n}{n^p}$ with x and p both real numbers. For what values of x and what values of p does this summation converge? *Last year:* 7/26 *correct*
(A) For $|x| < 1$, it converges for all $p \in \mathbb{R}$. For $|x| \geq 1$, it does not converge for any p.
(B) For $|x| \leq 1$, it converges for all $p \in \mathbb{R}$. For $|x| > 1$, it does not converge for any p.
(C) For $|x| < 1$, it converges for all $p \in \mathbb{R}$. For $|x| > 1$, it does not converge for any p. For $|x| = 1$, it converges if and only if $p > 1$.
(D) For $|x| < 1$, it converges for all $p \in \mathbb{R}$. For $|x| > 1$, it does not converge for any p. For $x = 1$, it converges if and only if $p > 0$. For $x = -1$, it converges if and only if $p > 1$.
(E) For $|x| < 1$, it converges for all $p \in \mathbb{R}$. For $|x| > 1$, it does not converge for any $p \in \mathbb{R}$. For $x = 1$, it converges if and only if $p > 1$. For $x = -1$, it converges if and only if $p > 0$.

Your answer: ____________________________

There is a result of calculus which states that, under suitable conditions, if $f_1, f_2, \dots, f_n, \dots$ are all functions, and we define $f(x) := \sum_{n=1}^\infty f_n(x)$, then $f'(x) = \sum_{n=1}^\infty f_n'(x)$. In other words, under suitable assumptions, we can differentiate a sum of countably many functions by differentiating each of them and adding up the derivatives.

We will not be going into what those assumptions are, but will consider some applications where you are explicitly told that these assumptions are satisfied.

(8) (**) Consider the summation $\zeta(p) := \sum_{n=1}^\infty \frac{1}{n^p}$ for $p > 1$. Assume that the required assumptions are valid for this summation, so that $\zeta'(p)$ is the sum of the derivatives of each of the terms (summands) with respect to p. What is the correct expression for $\zeta'(p)$? *Last year:* 8/26 *correct*
(A) $\sum_{n=1}^\infty \frac{-p}{n^{p+1}}$
(B) $\sum_{n=1}^\infty \frac{-1}{(p+1)n^{p+1}}$
(C) $\sum_{n=1}^\infty \frac{p}{n^{p-1}}$
(D) $\sum_{n=1}^\infty \frac{-\ln n}{n^p}$
(E) $\sum_{n=1}^\infty \frac{-\ln n}{n^{p+1}}$

Your answer: ____________________________

(9) Recall that we defined $F(x, p) := \sum_{n=1}^\infty \frac{x^n}{n^p}$ with x and p both real numbers. Assume that, for a particular fixed value of p, the summation satisfies the conditions as a function of x for $|x| < 1$. What is its derivative with respect to x, keeping p constant? *Last year:* 14/26 *correct*
(A) $\sum_{n=1}^\infty \frac{x^{n+1}}{n^{p+1}}$
(B) $\sum_{n=1}^\infty \frac{x^{n+1}}{n^{p-1}}$
(C) $\sum_{n=1}^\infty \frac{x^{n-1}}{n^{p+1}}$

(D) $\sum_{n=1}^{\infty} \frac{x^{n-1} \ln n}{n^{p+1}}$
(E) $\sum_{n=1}^{\infty} \frac{x^{n-1}}{n^{p-1}}$

Your answer: ____________________________

(10) The series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. Since it is a series of positive terms, this means that the partial sums get arbitrarily large. What is the approximate smallest value of N such that $\sum_{n=1}^{N} \frac{1}{n} > 100$? *Last year:* 14/26 *correct*
(A) Between 90 and 110
(B) Between 2000 and 3000
(C) Between 10^{40} and 10^{50}
(D) Between 10^{90} and 10^{110}
(E) Between 10^{220} and 10^{250}

Your answer: ____________________________