TAKE-HOME CLASS QUIZ: DUE FEBRUARY 8: LIMITS AT INFINITY AND IMPROPER INTEGRAL

MATH 153, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

YOU ARE FREE TO DISCUSS ALL QUESTIONS, BUT PLEASE MAKE SURE TO ONLY ENTER ANSWER CHOICES THAT YOU PERSONALLY ENDORSE

(1) If $\lim_{x\to\infty} f(x) = L$ for some finite L, this tells us that the graph of f has a:
(A) vertical asymptote
(B) horizontal asymptote
(C) vertical tangent
(D) horizontal tangent
(E) vertical cusp

Your answer: ______________________

(2) If $\lim_{x\to\infty} f(x) = L$ and $\lim_{x\to\infty} f'(x) = M$, where both L and M are finite, then:
(A) $L = 0$ but M need not be zero
(B) $M = 0$ but L need not be zero
(C) Both L and M must be zero.
(D) Neither L nor M need be zero.
(E) At least one of L and M must be zero, but it could be either one.

Your answer: ______________________

Consider the following $\epsilon - \delta$ definition of limit at ∞: $\lim_{x\to\infty} f(x) = L$ if for all $\epsilon > 0$, there exists $a \in \mathbb{R}$ such that for all $x > a$, $|f(x) - L| < \epsilon$.

(3) What is the smallest a that can be picked for the function $f = \arctan$ with L being its limit at ∞ and $\epsilon = \pi$?
(A) $\sqrt{3}$
(B) 1
(C) 0
(D) -1
(E) There is no smallest a. Any $a \in \mathbb{R}$ will do.

Your answer: ______________________

(4) What is the smallest a that can be picked for the function $f = \arctan$ with L being its limit at ∞ and $\epsilon = \pi/6$?
(A) $1/2$
(B) $1/\sqrt{3}$
(C) 1
(D) $\sqrt{3}$
(E) 2

Your answer: ______________________

(5) Suppose $f(x) := p(x)/q(x)$ is a rational function in reduced form (i.e., the numerator and denominator are relatively prime) and $\lim_{x\to c} f(x) = \infty$. Which of the following can you conclude about f?
(A) $x - c$ divides $p(x)$, and the largest r such that $(x-c)^r$ divides $p(x)$ is even.
(B) $x - c$ divides $q(x)$, and the largest r such that $(x-c)^r$ divides $q(x)$ is even.
(C) $x - c$ divides $p(x)$, and the largest r such that $(x-c)^r$ divides $p(x)$ is odd.
(D) $x - c$ divides $q(x)$, and the largest r such that $(x-c)^r$ divides $q(x)$ is odd.
(E) $x - c$ does not divide either $p(x)$ or $q(x)$.

Your answer: ____________________________

(6) Suppose $f(x) := p(x)/q(x)$ is a rational function in reduced form (i.e., the numerator and denominator are relatively prime) and $\lim_{x\to c^-} f(x) = \infty$ and $\lim_{x\to c^+} f(x) = -\infty$. Which of the following can you conclude about f?
(A) $x - c$ divides $p(x)$, and the largest r such that $(x-c)^r$ divides $p(x)$ is even.
(B) $x - c$ divides $q(x)$, and the largest r such that $(x-c)^r$ divides $q(x)$ is even.
(C) $x - c$ divides $p(x)$, and the largest r such that $(x-c)^r$ divides $p(x)$ is odd.
(D) $x - c$ divides $q(x)$, and the largest r such that $(x-c)^r$ divides $q(x)$ is odd.
(E) $x - c$ does not divide either $p(x)$ or $q(x)$.

Your answer: ____________________________

Suppose F is a function of two real variables, say x and t, so $F(x,t)$ is a real number for x and t restricted to suitable open intervals in the real number. Suppose, further, that F is jointly continuous (whatever that means) in x and t.

Define $f(t) := \int_0^\infty F(x,t)\,dx$. Here, while doing the integration, t is treated as a constant. x, the variable of integration, is being integrated on $[0,\infty)$.

Suppose further that f is defined and continuous for t in $(0,\infty)$.

In the next few questions, you are asked to compute the function f explicitly given the function F, for $t \in (0,\infty)$.

(7) $F(x,t) := e^{-tx}$. Find f.
(A) $f(t) = e^{-t}/t$
(B) $f(t) = e^{t}/t$
(C) $f(t) = 1/t$
(D) $f(t) = -1/t$
(E) $f(t) = -t$

Your answer: ____________________________

(8) $F(x,t) := 1/(t^2 + x^2)$. Find f.
(A) $f(t) = \pi/(2t)$
(B) $f(t) = \pi/t$
(C) $f(t) = 2\pi/t$
(D) $f(t) = \pi t$
(E) $f(t) = 2\pi t$

Your answer: ____________________________

(9) $F(x,t) := 1/(t^2 + x^2)^2$. Find f.
(A) $f(t) = \pi/t^3$
(B) $f(t) = \pi/(2t^3)$
(C) $f(t) = \pi/(4t^3)$
(D) $f(t) = \pi/(8t^3)$
(E) $f(t) = 3\pi/(8t^3)$

Your answer: ____________________________

(10) $F(x,t) = \exp(-(tx)^2)$. Use that $\int_0^\infty \exp(-x^2)\,dx = \sqrt{\pi}/2$. Find f.
(A) $f(t) = t^2\sqrt{\pi}/2$
(B) $f(t) = t\sqrt{\pi}/2$
(C) $f(t) = \sqrt{\pi}/2$
(D) $f(t) = \sqrt{\pi}/(2t)$
(E) $f(t) = \sqrt{\pi}/(2t^2)$

Your answer: ____________________________

(11) In the same general setup as above (but with none of these specific Fs), which of the following is a *sufficient* condition for f to be an increasing function of t?
(A) $t \mapsto F(x_0, t)$ is an increasing function of t for every choice of $x_0 \geq 0$.
(B) $x \mapsto F(x, t_0)$ is an increasing function of x for every choice of $t_0 \in (0, \infty)$.
(C) $t \mapsto F(x_0, t)$ is a decreasing function of t for every choice of $x_0 \geq 0$.
(D) $x \mapsto F(x, t_0)$ is a decreasing function of x for every choice of $t_0 \in (0, \infty)$.
(E) None of the above.

Your answer: ____________________________