CLASS QUIZ: FEBRUARY 3: DIFFERENTIAL EQUATIONS

MATH 153, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) It takes time T for $1/10$ of a radioactive substance to decay. How much does it take for $3/10$ of the same substance to decay? *Last yer:* 22/26 *correct*
   (A) Between T and $2T$
   (B) Between $2T$ and $3T$
   (C) Exactly $3T$
   (D) Between $3T$ and $4T$
   (E) Between $4T$ and $5T$

   Your answer: ____________________________

(2) (*) Suppose a function f satisfies the differential equation $f''(x) = 0$ for all $x \in \mathbb{R}$. Which of the following is true about $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$? *Last year:* 13/26 *correct*
   (A) If either limit is finite, then both are finite and they are equal. Otherwise, both the limits are infinities of opposite signs.
   (B) If either limit is finite, then both are finite and they are equal. Otherwise, both the limits are infinities of the same sign.
   (C) One of the limits is finite and the other is infinite.
   (D) Both the limits are finite and unequal.
   (E) Both the limits are infinite but they may be of the same or of opposite signs.

   Your answer: ____________________________

(3) (*) For y a function of x, consider the differential equation $(y')^2 - 3yy' + 2y^2 = 0$. What is the description of the **general solution** to this differential equation? *Last year:* 12/26 *correct*
   (A) $y = C_1e^x + C_2e^{2x}$, where C_1 and C_2 are arbitrary real numbers.
   (B) $y = C_1e^x + C_2e^{2x}$, where C_1 and C_2 are real numbers satisfying $C_1C_2 = 0$ (i.e., at least one of them is zero)
   (C) $y = C_1e^x + C_2e^{2x}$, where C_1 and C_2 are real numbers satisfying $C_1 + C_2 = 0$.
   (D) $y = C_1e^x + C_2e^{2x}$, where C_1 and C_2 are real numbers satisfying $C_1C_2 = 1$.
   (E) $y = C_1e^x + C_2e^{2x}$, where C_1 and C_2 are real numbers satisfying $C_1 + C_2 = 1$.

   Your answer: ____________________________

(4) (*) Suppose $F(t)$ represents the number of gigabytes of disk space that can be purchased with one dollar at time t in commercially available disk drive formats (not adjusted for inflation). Empirical observation shows that $F(1980) \approx 5 * 10^{-6}$, $F(1990) \approx 10^{-4}$, $F(2000) \approx 10^{-1}$, and $F(2010) \approx 10$. From these data, what is a good estimate for the "doubling time" of F, i.e., the time it takes for the number of gigabytes purchaseable with a dollar to double? *Last year:* 10/26 *correct*
   (A) Between 6 months and 1 year.
   (B) Between 1 year and 2 years.
   (C) Between 2 years and 4 years.
   (D) Between 4 years and 5 years.
   (E) Between 5 years and 6 years.

   Your answer: ____________________________

(5) (*) The size S of an online social network satisfies the differential equation $S'(t) = kS(t)(1 - (S(t))/(W(t)))$ where $W(t)$ is the world population at time t. Suppose $W(t)$ itself satisfies the differential equation $W'(t) = k_0 W(t)$ where k_0 is positive but much smaller than k. How would we expect S to behave, assuming that initially, $S(t)$ is positive but much smaller than $W(t)$? *Last year:* 11/26 *correct*

(A) It initially appears like an exponential function with exponential growth rate k, but over time, it slows down to (roughly) an exponential function with exponential growth rate k_0.
(B) It initially appears like an exponential function with exponential growth rate k_0, but over time, it speeds up to (roughly) an exponential function with exponential growth rate k.
(C) It behaves roughly like an exponential function with growth rate k_0 for all time.
(D) It behaves roughly like an exponential function with growth rate k for all time.
(E) It initially behaves like an exponential function with exponential growth rate k but then it starts declining.

Your answer: ____________________________

(6) (**) Suppose the growth of a population P with time is described by the equation $dP/dt = aP^{1-\beta}$ with $a > 0$ and $0 < \beta < 1$. What can we say about the nature of the population as a function of t, assuming that the population at time 0 is positive? *Last year:* 8/26 *correct*

(A) The population grows as a sub-linear power function of t, i.e., roughly like t^γ where $0 < \gamma < 1$.
(B) The population grows as a linear power function of t, i.e., roughly like t.
(C) The population grows as a superlinear power function of t, i.e., roughly like t^γ where $\gamma > 1$.
(D) The population grows like an exponential function of t, i.e., roughly like e^{kt} for some $k > 0$.
(E) The population grows super-exponentially, i.e., it eventually surpasses any exponential function.

Your answer: ____________________________

(7) (**) Suppose the growth of a population P with time is described by the equation $dP/dt = aP^{1+\theta}$ with $0 < \theta$ *and* $a > 0$. What can we say about the nature of the population as a function of t, assuming that the population at time 0 is positive? *Last year:* 3/26 *correct*

(A) The population approaches infinity in finite time, and the differential equation makes no sense beyond that.
(B) The population increases at a decreasing rate and approaches a horizontal asymptote, i.e., it proceeds to a finite limit as time approaches infinity.
(C) The population grows linearly.
(D) The population grows super-linearly but sub-exponentially.
(E) The population grows exponentially.

Your answer: ____________________________

(8) (**) Let $r(t)$ denote the fractional growth rate per annum in per capita income, which we denote by $I(t)$. In other words, $r(t) = I'(t)/I(t)$, measured in units of (per year). It is observed that, over a certain time period, $r'(t) = kr(t)$ for a positive constant k. Assuming that the initial values of $I(t)$ and $r(t)$ are positive, what best describes the nature of the function $I(t)$? *Last year:* 2/26 *correct*

(A) $I(t)$ is a linear function of t, i.e., per capita income is getting incremented by a constant *amount* (rather than a constant proportion).
(B) $I(t)$ is a super-linear but sub-exponential function of t, i.e., per capita income is rising, but less than exponentially.
(C) $I(t)$ is an exponential function of t, i.e., per capita income is rising by a constant proportion per year.
(D) $I(t)$ is a super-exponential function of t but slower than a doubly exponential function of t.
(E) $I(t)$ is a doubly exponential function of t.

Your answer: ____________________________