CLASS QUIZ: SEPTEMBER 26; TOPIC: FUNCTIONS

VIPUL NAIK

Your name (print clearly in capital letters): ____

Write your answer in the space provided. In the space below, you can explain your work if you want (this will not affect scoring). I may or may not get time to look at the work you have done, but it may help you recall how you arrived at a particular answer.

Note: The difficulty level of this quiz is *not* intended to be representative of the difficulty level of quizzes for this course. Today's quiz is meant as a first-day warm-up. If you're having trouble with these questions, this course level may be too advanced for you.

(1) Consider the function $f(x) := |x+1| - |x|$. For which of the following values of x is $f(x)$ equal to 0?
(A) $-\frac{1}{2}$
(B) $-\frac{1}{3}$
(C) 0
(D) $\frac{1}{3}$
(E) $\frac{1}{2}$

Your answer: ____________________________

(2) Consider the function $f(x) := x^2 + 1$. What is the polynomial describing $f(f(x))$?
(A) $x^2 + 2$
(B) $x^4 + x^2 + 1$
(C) $x^4 + x^2 + 2$
(D) $x^4 + 2x^2 + 1$
(E) $x^4 + 2x^2 + 2$

Your answer: ____________________________

(3) Consider the function $f(x) := \frac{x}{x^2+1}$. What is $f(f(1))$?
(A) 1/5
(B) 2/5
(C) 4/5
(D) 5/4
(E) 5/8

Your answer: ____________________________

(4) Consider the function $f(x) := x + 1$. What is $f(f(x))$?
(A) x
(B) $x + 2$
(C) $2x + 1$
(D) $(x + 1)^2$
(E) $x^2 + 1$

Your answer: ______________________________

(5) If a circle has radius r, the area of the circle is πr^2. What is the area of a circle with diameter d?
(A) $\pi d^2/4$
(B) $\pi d^2/2$
(C) πd^2
(D) $2\pi d^2$
(E) $4\pi d^2$

Your answer: ______________________________

CLASS QUIZ: SEPTEMBER 28; TOPIC: FUNCTIONS

VIPUL NAIK

Your name (print clearly in capital letters): ______________________________________

Write your answer in the space provided. In the space below, you can explain your work if you want (this will not affect scoring). I may or may not get time to look at the work you have done, but it may help you recall how you arrived at a particular answer.

You are expected to take about one minute per question.

Questions marked with a (*) are questions that are somewhat trickier, with the probability of getting the question correct being about 50% or less. For these questions, you are free to discuss the questions with others while making your attempt.

(1) Suppose f and g are functions from $\mathbb{R}$ to $\mathbb{R}$. Suppose both f and g are even, i.e., $f(x) = f(-x)$ for all $x \in \mathbb{R}$ and $g(x) = g(-x)$ for all $x \in \mathbb{R}$. Which of the following is *not* guaranteed to be an even function from the given information? *Last year's performance*: 11/15 correct

Note: For this question, it is possible to solve the question by taking a few simple examples. You're free to do this, but the recommended method for tackling the question is to handle it **abstractly**, *i.e., try to prove or disprove in general for each function whether it is even.*

(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
(B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$
(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(E) None of the above, i.e., they are all guaranteed to be even functions.

Your answer: ___________________________

(2) (*) Suppose f and g are functions from $\mathbb{R}$ to $\mathbb{R}$. Suppose both f and g are odd, i.e., $f(-x) = -f(x)$ for all $x \in \mathbb{R}$ and $g(-x) = -g(x)$ for all $x \in \mathbb{R}$. Which of the following is *not* guaranteed to be an odd function from the given information? *Last year's performance*: 7/15 correct

Note: For this question, it is possible to solve the question by taking a few simple examples. You're free to do this, but the recommended method for tackling the question is to handle it **abstractly**, *i.e., try to prove or disprove in general for each function whether it is odd.*

(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
(B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$
(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(E) None of the above, i.e., they are all guaranteed to be odd functions.

Your answer: ___________________________

(3) For which of the following pairs of polynomial functions f and g is it true that $f \circ g \neq g \circ f$? *Last year's performance*: 14/15 correct
(A) $f(x) := x^2$ and $g(x) := x^3$
(B) $f(x) := x + 1$ and $g(x) := x + 2$
(C) $f(x) := x^2 + 1$ and $g(x) := x^2 + 1$
(D) $f(x) := -x$ and $g(x) := x^2$
(E) $f(x) := -x$ and $g(x) := x^3$

Your answer: ____________________________

(4) (*) Which of the following functions is *not* periodic? *Last year's performance*: 8/15 correct
(A) $\sin(x^2)$
(B) $\sin^2 x$
(C) $\sin(\sin x)$
(D) $\sin(x + 13)$
(E) $(\sin x) + 13$

Your answer: ____________________________

(5) (*) What is the domain of the function $\sqrt{1-x} + \sqrt{x-2}$? Here, domain refers to the *largest subset of the reals* on which the function can be defined. *Last year's performance*: 8/15 correct

Hint: Think clearly, first about what the domain of each of the two functions being added is, and then about whether you need to take the union or the intersection of the domains of the individual functions.
(A) $(1, 2)$
(B) $[1, 2]$
(C) $(-\infty, 1) \cup (2, \infty)$
(D) $(-\infty, 1] \cup [2, \infty)$
(E) None of the above

Your answer: ____________________________

CLASS QUIZ: SEPTEMBER 30: LIMITS

VIPUL NAIK

Your name (print clearly in capital letters): ____

Write your answer in the space provided. In the space below, you can explain your work if you want (this will not affect scoring). I may or may not get time to look at the work you have done, but it may help you recall how you arrived at a particular answer.

You are expected to take about one minute per question.

Questions marked with a (*) are questions that are somewhat trickier, with the probability of getting the question correct being about 50% or less. For these questions, you are free to discuss the questions with others while making your attempt.

Questions marked with a (**) are questions where, in a previous administration of this quiz, a specific incorrect option was chosen by as many people as or more people than the correct option. For these questions, you are free to discuss the questions with others while making your attempt.

(1) (**) We call a function f left continuous on an open interval I if, for all $a \in I$, $\lim_{x \to a^-} f(x) = f(a)$. Which of the following is an example of a function that is left continuous but not continuous on $(0,1)$? *Last year's performance*: 6/13 correct

(A) $f(x) := \{ \begin{array}{rl} x, & 0 < x \le 1/2 \\ 2x, & 1/2 < x < 1 \end{array}$

(B) $f(x) := \{ \begin{array}{rl} x, & 0 < x < 1/2 \\ 2x, & 1/2 \le x < 1 \end{array}$

(C) $f(x) := \{ \begin{array}{rl} x, & 0 < x \le 1/2 \\ 2x - (1/2), & 1/2 < x < 1 \end{array}$

(D) $f(x) := \{ \begin{array}{rl} x, & 0 < x < 1/2 \\ 2x - (1/2), & 1/2 \le x < 1 \end{array}$

(E) All of the above

Your answer: ____________________________

(2) (**) Suppose f and g are functions $(0,1)$ to $(0,1)$ that are both left continuous on $(0,1)$. Which of the following is *not* guaranteed to be left continuous on $(0,1)$? *Last year's performance*: 4/13 correct

(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$

(B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$

(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$

(D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$

(E) None of the above, i.e., they are all guaranteed to be left continuous functions

Your answer: ____________________________

(3) (*) Consider the function:

$$f(x) := \{ \begin{array}{rl} x, & x \text{ rational} \\ 1/x, & x \text{ irrational} \end{array}$$

What is the set of all points at which f is continuous? *Last year's performance*: 5/13 correct

(A) $\{0, 1\}$
(B) $\{-1, 1\}$
(C) $\{-1, 0\}$
(D) $\{-1, 0, 1\}$
(E) f is continuous everywhere

Your answer: ____________________________

CLASS QUIZ: OCTOBER 3: LIMITS

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) Which of these is the correct interpretation of $\lim_{x \to c} f(x) = L$ in terms of the definition of limit? *Last year's performance:* 9/12 *correct*
  (A) For every $\alpha > 0$, there exists $\beta > 0$ such that if $0 < |x - c| < \alpha$, then $|f(x) - L| < \beta$.
  (B) There exists $\alpha > 0$ such that for every $\beta > 0$, and $0 < |x - c| < \alpha$, we have $|f(x) - L| < \beta$.
  (C) For every $\alpha > 0$, there exists $\beta > 0$ such that if $0 < |x - c| < \beta$, then $|f(x) - L| < \alpha$.
  (D) There exists $\alpha > 0$ such that for every $\beta > 0$ and $0 < |x - c| < \beta$, we have $|f(x) - L| < \alpha$.
  (E) None of the above

  Your answer: ____________________________

(2) Suppose $f : \mathbb{R} \to \mathbb{R}$ is a function. Which of the following says that f does not have a limit at any point in $\mathbb{R}$ (i.e., there is no point $c \in \mathbb{R}$ for which $\lim_{x \to c} f(x)$ exists)? *Last year's performance:* 10/12 *correct*
  (A) For every $c \in \mathbb{R}$, there exists $L \in \mathbb{R}$ such that for every $\epsilon > 0$, there exists $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, we have $|f(x) - L| \geq \epsilon$.
  (B) There exists $c \in \mathbb{R}$ such that for every $L \in \mathbb{R}$, there exists $\epsilon > 0$ such that for every $\delta > 0$, there exists x satisfying $0 < |x - c| < \delta$ and $|f(x) - L| \geq \epsilon$.
  (C) For every $c \in \mathbb{R}$ and every $L \in \mathbb{R}$, there exists $\epsilon > 0$ such that for every $\delta > 0$, there exists x satisfying $0 < |x - c| < \delta$ and $|f(x) - L| \geq \epsilon$.
  (D) There exists $c \in \mathbb{R}$ and $L \in \mathbb{R}$ such that for every $\epsilon > 0$, there exists $\delta > 0$ such that for all x satisfying $0 < |x - c| < \delta$, we have $|f(x) - L| \geq \epsilon$.
  (E) All of the above.

  Your answer: ____________________________

(3) In the usual $\epsilon - \delta$ definition of limit for a given limit $\lim_{x \to c} f(x) = L$, if a given value $\delta > 0$ works for a given value $\epsilon > 0$, then which of the following is true? *Last year's performance:* 17/26 *correct, appeared in 153 quiz*
  (A) Every smaller positive value of δ works for the same ϵ. Also, the given value of δ works for every smaller positive value of ϵ.
  (B) Every smaller positive value of δ works for the same ϵ. Also, the given value of δ works for every larger value of ϵ.
  (C) Every larger value of δ works for the same ϵ. Also, the given value of δ works for every smaller positive value of ϵ.
  (D) Every larger value of δ works for the same ϵ. Also, the given value of δ works for every larger value of ϵ.
  (E) None of the above statements need always be true.

  Your answer: ____________________________

(4) Which of the following is a correct formulation of the statement $\lim_{x\to c} f(x) = L$, in a manner that avoids the use of ϵs and δs? *Not appeared in previous years*

(A) For every open interval centered at c, there is an open interval centered at L such that the image under f of the open interval centered at c (excluding the point c itself) is contained in the open interval centered at L.

(B) For every open interval centered at c, there is an open interval centered at L such that the image under f of the open interval centered at c (excluding the point c itself) contains the open interval centered at L.

(C) For every open interval centered at L, there is an open interval centered at c such that the image under f of the open interval centered at c (excluding the point c itself) is contained in the open interval centered at L.

(D) For every open interval centered at L, there is an open interval centered at c such that the image under f of the open interval centered at c (excluding the point c itself) contains the open interval centered at L.

(E) None of the above.

Your answer: ____________________________

CLASS QUIZ: OCTOBER 7: LIMIT THEOREMS

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

Questions marked with a (*) are questions that are somewhat trickier, with the probability of getting the question correct being about 50% or less. For these questions, you are free to discuss the questions with others while making your attempt.

Questions marked with a (**) are questions where, in a previous administration of this quiz, a specific incorrect option was chosen by as many people as or more people than the correct option. For these questions, you are free to discuss the questions with others while making your attempt.

(1) (**) Which of the following statements is **always true**? *Last year's performance:* 2/11 *correct*
  (A) The range of a continuous nonconstant function on a closed bounded interval (i.e., an interval of the form $[a, b]$) is a closed bounded interval (i.e., an interval of the form $[m, M]$).
  (B) The range of a continuous nonconstant function on an open bounded interval (i.e., an interval of the form (a, b)) is an open bounded interval (i.e., an interval of the form (m, M)).
  (C) The range of a continuous nonconstant function on a closed interval that may be bounded or unbounded (i.e., an interval of the form $[a, b]$, $[a, \infty)$, $(-\infty, a]$, or $(-\infty, \infty)$) is also a closed interval that may be bounded or unbounded.
  (D) The range of a continuous nonconstant function on an open interval that may be bounded or unbounded (i.e., an interval of the form (a, b),(a, ∞), $(-\infty, a)$, or $(-\infty, \infty)$), is also an open interval that may be bounded or unbounded.
  (E) None of the above.

  Your answer: ____________________________

(2) (**) Suppose $g : \mathbb{R} \to \mathbb{R}$ is a continuous function such that $\lim_{x \to 0} g(x)/x = A$ for some constant $A \neq 0$. What is $\lim_{x \to 0} g(g(x))/x$? *Last year's performance:* 1/12 *correct*
  (A) 0
  (B) A
  (C) A^2
  (D) $g(A)$
  (E) $g(A)/A$

  Your answer: ____________________________

(3) Suppose $I = (a, b)$ is an open interval. A function $f : I \to \mathbb{R}$ is termed *piecewise continuous* if there eixst points $a_0 < a_1 < a_2 < \cdots < a_n$ (dependent on f) with $a = a_0$ and $a_n = b$, such that f is continuous on each interval (a_i, a_{i+1}). In other words, f is continuous at every point in (a, b) except possibly the a_is.

  Suppose f and g are piecewise continuous functions on the same interval I (with possibly different sets of a_is). Which of the following is/are guaranteed to be piecewise continuous on I? *Last year's performance:* 9/11 *correct*
  (A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
  (B) $f - g$, i.e., the function $x \mapsto f(x) - g(x)$

(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(D) All of the above
(E) None of the above

Your answer: ____________________________

(4) Suppose f and g are everywhere defined and $\lim_{x\to 0} f(x) = 0$. Which of these pieces of information is **not sufficient** to conclude that $\lim_{x\to 0} f(x)g(x) = 0$? *Last year's performance:* 8/11 *correct*
(A) $\lim_{x\to 0} g(x) = 0$.
(B) $\lim_{x\to 0} g(x)$ is a constant not equal to zero.
(C) There exists $\delta > 0$ and $B > 0$ such that for $0 < |x| < \delta$, $|g(x)| < B$.
(D) $\lim_{x\to 0} g(x) = \infty$, i.e., for every $N > 0$ there exists $\delta > 0$ such that if $0 < |x| < \delta$, then $g(x) > N$.
(E) None of the above, i.e., they are all sufficient to conclude that $\lim_{x\to 0} f(x)g(x) = 0$.

Your answer: ____________________________

(5) f and g are functions defined for all real values. c is a real number. Which of these statements is ***not* necessarily true**? *Last year's performance:* 9/11 *correct*
(A) If $\lim_{x\to c^-} f(x) = L$ and $\lim_{x\to c^-} g(x) = M$, then $\lim_{x\to c^-}(f(x) + g(x))$ exists and is equal to $L + M$.
(B) If $\lim_{x\to c^-} g(x) = L$ and $\lim_{x\to L^-} f(x) = M$, then $\lim_{x\to c^-} f(g(x)) = M$.
(C) If there exists an open interval containing c on which f is continuous and there exists an open interval containing c on which g is continuous, then there exists an open interval containing c on which $f + g$ is continuous.
(D) If there exists an open interval containing c on which f is continuous and there exists an open interval containing c on which g is continuous, then there exists an open interval containing c on which the product $f \cdot g$ (i.e., the function $x \mapsto f(x)g(x)$) is continuous.
(E) None of the above, i.e., they are all necessarily true.

Your answer: ____________________________

CLASS QUIZ: OCTOBER 10: DERIVATIVES

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

Write your answer in the space provided. In the space below, you can explain your work if you want (this will not affect scoring). I may or may not get time to look at the work you have done, but it may help you recall how you arrived at a particular answer.

You are expected to take about one minute per question.

(1) Consider the expression $x^2 + t^2 + xt$. What is the derivative of this with respect to x (with t assumed to be a constant)? *Last year:* 11/12 *correct*
   (A) $2x + 2t + x + t$
   (B) $2x + 2t + 1$
   (C) $2x + 2t$
   (D) $2x + t + 1$
   (E) $2x + t$

   Your answer: ______________________

(2) Which of the following verbal statements is **not valid as a general rule**? *Last year:* 10/12 *correct*
   (A) The derivative of the sum of two functions is the sum of the derivatives of the functions.
   (B) The derivative of the difference of two functions is the difference of the derivatives of the functions.
   (C) The derivative of a constant times a function is the same constant times the derivative of the function.
   (D) The derivative of the product of two functions is the product of the derivatives of the functions.
   (E) None of the above, i.e., they are all valid as general rules.

   Your answer: ______________________

PLEASE TURN OVER FOR THE THIRD AND FOURTH QUESTION.

(3) (*) Which of the following statements is **definitely true** about the tangent line to the graph of an everywhere differentiable function f on $\mathbb{R}$ at the point $(a, f(a))$ (Here, "everywhere differentiable" means that the derivative of f is defined and finite for all $x \in \mathbb{R}$)? *Last year:* 6/12 *correct*

(A) The tangent line intersects the curve at precisely one point, namely $(a, f(a))$.
(B) The tangent line intersects the x-axis.
(C) The tangent line intersects the $f(x)$-axis (the y-axis).
(D) Any line through $(a, f(a))$ other than the tangent line intersects the graph of f at at least one other point.
(E) None of the above need be true.

Your answer: ____________________________

(4) (*) For a function $f : (0, \infty) \to (0, \infty)$, denote by $f^{(k)}$ the k^{th} derivative of f. Suppose $f(x) := x^r$ with domain $(0, \infty)$, and r a rational number. What is the **precise set of values** of r satisfying the following: there exist a positive integer k (dependent on r) for which $f^{(k)}$ is identically the zero function. *Last year:* 4/12 *correct*

(A) r should be an integer.
(B) r should be a nonnegative integer.
(C) r should be a positive integer.
(D) r should be a nonnegative rational number.
(E) r should be a positive rational number.

Your answer: ____________________________

CLASS QUIZ: OCTOBER 12: DERIVATIVES

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

Write your answer in the space provided. In the space below, you can explain your work if you want (this will not affect scoring). I may or may not get time to look at the work you have done, but it may help you recall how you arrived at a particular answer.

You are expected to take about one minute per question.

(1) (**) Suppose f is a differentiable function on $\mathbb{R}$. Which of the following implications is **false**? *Last year:* 0/14 *correct*
  (A) If f is even, then f' is odd.
  (B) If f is odd, then f' is even.
  (C) If f' is even, then f is odd.
  (D) If f' is odd, then f is even.
  (E) None of the above, i.e., they are all true.

  Your answer: ____________________________

(2) (*) A function f on $\mathbb{R}$ is said to satisfy the *intermediate value property* if, for any $a < b \in \mathbb{R}$, and any d between $f(a)$ and $f(b)$, there exists $c \in [a, b]$ such that $f(c) = d$. Which (one or more) of the following functions satisfies the intermediate value property? *Last year:* 7/14 *correct*
  (A) $f(x) := \{ \begin{array}{rl} \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{array}$
  (B) $f(x) := \{ \begin{array}{ll} 1, & x \text{ rational} \\ 0, & x \text{ irrational} \end{array}$
  (C) $f(x) := \{ \begin{array}{ll} x, & x \text{ rational} \\ 0, & x \text{ irrational} \end{array}$
  (D) All of the above
  (E) None of the above

  Your answer: ____________________________

PLEASE TURN OVER FOR THE THIRD AND FOURTH QUESTION.

(3) Which (one or more) of the following functions have a period of π? *Last year:* 12/14 *correct*

(A) $x \mapsto \sin^2 x$
(B) $x \mapsto |\sin x|$
(C) $x \mapsto \cos^2 x$
(D) $x \mapsto |\cos x|$
(E) All of the above

Your answer: ____________________________

(4) Suppose f is a function defined on all of $\mathbb{R}$ such that f' is a periodic function defined on all of $\mathbb{R}$. What can we conclude is **definitely true** about f? *Last year:* 8/14 *correct*

(A) f must be a linear function.
(B) f must be a periodic function.
(C) f can be expressed as the sum of a linear and a periodic function, but f need not be either linear or periodic.
(D) f can be expressed as the product of a linear and periodic function, but f need not be either linear or periodic.
(E) f can be expressed as a composite of a linear and a periodic function, but f need not be either linear or periodic.

Your answer: ____________________________

CLASS QUIZ: OCTOBER 14: DERIVATIVES

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) Suppose f and g are functions from $\mathbb{R}$ to $\mathbb{R}$ that are everywhere differentiable. Which of the following functions is/are guaranteed to be everywhere differentiable? *Last year:* 13/14 *correct*
   (A) $f + g$
   (B) $f - g$
   (C) $f \cdot g$
   (D) $f \circ g$
   (E) All of the above

   Your answer: ____________________________

(2) Suppose f and g are both twice differentiable functions everywhere on $\mathbb{R}$. Which of the following is the correct formula for $(f \cdot g)''$? *Last year:* 13/14 *correct*
   (A) $f'' \cdot g + f \cdot g''$
   (B) $f'' \cdot g + f' \cdot g' + f \cdot g''$
   (C) $f'' \cdot g + 2f' \cdot g' + f \cdot g''$
   (D) $f'' \cdot g - f' \cdot g' + f \cdot g''$
   (E) $f'' \cdot g - 2f' \cdot g' + f \cdot g''$

   Your answer: ____________________________

PLEASE TURN OVER FOR THE THIRD AND FOURTH QUESTION.

(3) Suppose f and g are both twice differentiable functions everywhere on $\mathbb{R}$. Which of the following is the correct formula for $(f \circ g)''$? *Last year:* 14/14 *correct*

(A) $(f'' \circ g) \cdot g''$
(B) $(f'' \circ g) \cdot (f' \circ g') \cdot g''$
(C) $(f'' \circ g) \cdot (f' \circ g') \cdot (f \circ g'')$
(D) $(f'' \circ g) \cdot (g')^2 + (f' \circ g) \cdot g''$
(E) $(f' \circ g') \cdot (f \circ g) + (f'' \circ g'')$

Your answer: ____________________________

(4) Suppose f is an everywhere differentiable function on $\mathbb{R}$ and $g(x) := f(x^3)$. What is $g'(x)$? *Last year:* 13/14 *correct*

(A) $3x^2 f(x)$
(B) $3x^2 f'(x)$
(C) $3x^2 f(x^3)$
(D) $3x^2 f'(x^3)$
(E) $f'(3x^2)$

Your answer: ____________________________

CLASS QUIZ: OCTOBER 19: INCREASE/DECREASE AND MAXIMA/MINIMA

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____
You are expected to take about one minute per question.

(1) (*) Suppose f is a function defined on a closed interval $[a, c]$. Suppose that the left-hand derivative of f at c exists and equals ℓ. Which of the following implications is **true in general**? *Last year:* 8/15 *correct*
   (A) If $f(x) < f(c)$ for all $a \le x < c$, then $\ell < 0$.
   (B) If $f(x) \le f(c)$ for all $a \le x < c$, then $\ell \le 0$.
   (C) If $f(x) < f(c)$ for all $a \le x < c$, then $\ell > 0$.
   (D) If $f(x) \le f(c)$ for all $a \le x < c$, then $\ell \ge 0$.
   (E) None of the above is true in general.

   Your answer: ____________________________

(2) (**) Suppose f and g are increasing functions from $\mathbb{R}$ to $\mathbb{R}$. Which of the following functions is *not* guaranteed to be an increasing function from $\mathbb{R}$ to $\mathbb{R}$? *Last year:* 1/15 *correct*
   (A) $f + g$
   (B) $f \cdot g$
   (C) $f \circ g$
   (D) All of the above, i.e., none of them is guaranteed to be increasing.
   (E) None of the above, i.e., they are all guaranteed to be increasing.

   Your answer: ____________________________

PLEASE TURN OVER FOR THE THIRD AND FOURTH QUESTION.

(3) (**) Suppose f is a continuous function defined on an open interval (a, b) and c is a point in (a, b). Which of the following implications is **true**? *Last year:* 5/15 *correct*

(A) If c is a point of local minimum for f, then there is a value $\delta > 0$ and an open interval $(c - \delta, c + \delta) \subseteq (a, b)$ such that f is non-increasing on $(c - \delta, c)$ and non-decreasing on $(c, c + \delta)$.
(B) If there is a value $\delta > 0$ and an open interval $(c - \delta, c + \delta) \subseteq (a, b)$ such that f is non-increasing on $(c - \delta, c)$ and non-decreasing on $(c, c + \delta)$, then c is a point of local minimum for f.
(C) If c is a point of local minimum for f, then there is a value $\delta > 0$ and an open interval $(c - \delta, c + \delta) \subseteq (a, b)$ such that f is non-decreasing on $(c - \delta, c)$ and non-increasing on $(c, c + \delta)$.
(D) If there is a value $\delta > 0$ and an open interval $(c - \delta, c + \delta) \subseteq (a, b)$ such that f is non-decreasing on $(c - \delta, c)$ and non-increasing on $(c, c + \delta)$, then c is a point of local minimum for f.
(E) All of the above are true.

Your answer: ____________________________

(4) (**) Suppose f is a continuously differentiable function on $\mathbb{R}$ and f' is a periodic function with period h. (Recall that periodic derivative implies that the original function is a sum of ...). Suppose S is the set of points of local maximum for f, and T is the set of local maximum values. Which of the following is **true in general** about the sets S and T? *Last year:* 3/15 *correct*

(A) The set S is invariant under translation by h (i.e., $x \in S$ if and only if $x + h \in S$) and all the values in the set T are in the image of the set $[0, h]$ under f.
(B) The set S is invariant under translation by h (i.e., $x \in S$ if and only if $x + h \in S$) but all the values in the set T need not be in the image of the set $[0, h]$ under f.
(C) Both the sets S and T are invariant under translation by h.
(D) Both the sets S and T are finite.
(E) Both the sets S and T are infinite.

Your answer: ____________________________

CLASS QUIZ: OCTOBER 21: MAX-MIN PROBLEMS

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____
Note: Questions 5-7 have only four options.

(1) Consider all the rectangles with perimeter equal to a fixed length $p > 0$. Which of the following **is true** for the unique rectangle which is a square, compared to the other rectangles? *Last year:* 15/15 *correct*
 (A) It has the largest area and the largest length of diagonal.
 (B) It has the largest area and the smallest length of diagonal.
 (C) It has the smallest area and the largest length of diagonal.
 (D) It has the smallest area and the smallest length of diagonal.
 (E) None of the above.

 Your answer: ____________________________

(2) Suppose the total perimeter of a square and an equilateral triangle is L. (We can choose to allocate all of L to the square, in which case the equilateral triangle has side zero, and we can choose to allocate all of L to the equilateral triangle, in which case the square has side zero). Which of the following statements **is true** for the sum of the areas of the square and the equilateral triangle? (The area of an equilateral triangle is $\sqrt{3}/4$ times the square of the length of its side). *Last year:* 9/15 *correct*
 (A) The sum is minimum when all of L is allocated to the square.
 (B) The sum is maximum when all of L is allocated to the square.
 (C) The sum is minimum when all of L is allocated to the equilateral triangle.
 (D) The sum is maximum when all of L is allocated to the equilateral triangle.
 (E) None of the above.

 Your answer: ____________________________

(3) Suppose x and y are positive numbers such as $x + y = 12$. For **what values** of x and y is x^2y maximum? *Last year:* 12/15 *correct*
 (A) $x = 3$, $y = 9$
 (B) $x = 4$, $y = 8$
 (C) $x = 6$, $y = 6$
 (D) $x = 8$, $y = 4$
 (E) $x = 9$, $y = 3$

 Your answer: ____________________________

(4) (**) Consider the function $p(x) := x^2+bx+c$, with x restricted to *integer inputs*. Suppose b and c are integers. The minimum value of p is attained either at a single integer or at two consecutive integers. Which of the following is a **sufficient condition** for the minimum to occur at two consecutive integers? *Last year:* 4/15 *correct*

(A) b is odd
(B) b is even
(C) c is odd
(D) c is even
(E) None of these conditions is sufficient.

Your answer: ____________________________

(5) (**) Consider a hollow cylinder with no top and bottom and total curved surface area S. What can we say about the **maximum and minimum** possible values of the **volume**? (for radius r and height h, the curved surface area is $2\pi rh$ and the volume is $\pi r^2 h$). *Last year:* 6/15 *correct*

(A) The volume can be made arbitrarily small (i.e., as close to zero as we desire) and arbitrarily large (i.e., as large as we want).
(B) There is a positive minimum value for the volume, but it can be made arbitrarily large.
(C) There is a finite maximum value for the volume, but it can be made arbitrarily small.
(D) There is both a finite positive minimum and a finite positive maximum for the volume.

Your answer: ____________________________

(6) Consider a hollow cylinder with a bottom but no top and total surface area (curved surface plus bottom) S. What can we say about the **maximum and minimum** possible values of the **volume**? (for radius r and height h, the curved surface area is $2\pi rh$ and the volume is $\pi r^2 h$). *Last year:* 8/15 *correct*

(A) The volume can be made arbitrarily small (i.e., as close to zero as we desire) and arbitrarily large (i.e., as large as we want).
(B) There is a positive minimum value for the volume, but it can be made arbitrarily large.
(C) There is a finite maximum value for the volume, but it can be made arbitrarily small.
(D) There is both a finite positive minimum and a finite positive maximum for the volume.

Your answer: ____________________________

(7) (**) Consider a hollow cylinder with a bottom and a top and total surface area (curved surface plus bottom and top) S. What can we say about the **maximum and minimum** possible values of the **volume**? (for radius r and height h, the curved surface area is $2\pi rh$ and the volume is $\pi r^2 h$). *Last year:* 5/15 *correct*

(A) The volume can be made arbitrarily small (i.e., as close to zero as we desire) and arbitrarily large (i.e., as large as we want).
(B) There is a positive minimum value for the volume, but it can be made arbitrarily large.
(C) There is a finite maximum value for the volume, but it can be made arbitrarily small.
(D) There is both a finite positive minimum and a finite positive maximum for the volume.

Your answer: ____________________________

CLASS QUIZ: OCTOBER 24: CONCAVE, INFLECTIONS, TANGENTS, CUSPS, ASYMPTOTES

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) Consider the function $f(x) := x^3(x-1)^4(x-2)^2$. Which of the following **is true**? *Last year:* 11/15 *correct*
(A) 0, 1, and 2 are all critical points and all of them are points of local extrema.
(B) 0, 1, and 2 are all critical points, but only 0 is a point of local extremum.
(C) 0, 1, and 2 are all critical points, but only 1 and 2 are points of local extrema.
(D) 0, 1, and 2 are all critical points, and none of them is a point of local extremum.
(E) 1 and 2 are the only critical points.

Your answer: ____________________________

(2) Suppose f and g are continuously differentiable functions on $\mathbb{R}$. Suppose f and g are both concave up. Which of the following is **always true**? *Last year:* 8/15 *correct*
(A) $f+g$ is concave up.
(B) $f-g$ is concave up.
(C) $f \cdot g$ is concave up.
(D) $f \circ g$ is concave up.
(E) All of the above.

Your answer: ____________________________

(3) Consider the function $p(x) := x(x-1)\dots(x-n)$, where $n \geq 1$ is a positive integer. How many points of inflection does p have? *Last year:* 7/15 *correct*
(A) $n-3$
(B) $n-2$
(C) $n-1$
(D) n
(E) $n+1$

Your answer: ____________________________

(4) Suppose f is a polynomial function of degree $n \geq 2$. What can you say about the sense of concavity of the function f for **large enough inputs**, i.e., as $x \to +\infty$? (Note that if $n \leq 1$, f is linear so we do not have concavity in either sense). *Last year:* 12/15 *correct*
(A) f is eventually concave up.
(B) f is eventually concave down.

(C) f is eventually either concave up or concave down, and which of these cases occurs depends on the sign of the leading coefficient of f.
(D) f is eventually either concave up or concave down, and which of these cases occurs depends on whether the degree of f is even or odd.
(E) f may be concave up, concave down, or neither.

Your answer: ____________________________

(5) (**) Suppose f is a continuously differentiable function on $[a, b]$ and f' is continuously differentiable at all points of $[a, b]$ except an interior point c, where it has a vertical cusp. What can we say is **definitely true** about the behavior of f at c? *Last year:* 3/15 *correct*
(A) f attains a local extreme value at c.
(B) f has a point of inflection at c.
(C) f has a critical point at c that does not correspond to a local extreme value.
(D) f has a vertical tangent at c.
(E) f has a vertical cusp at c.

Your answer: ____________________________

(6) (**) Suppose f and g are continuous functions on $\mathbb{R}$, such that f attains a vertical tangent at a and is continuously differentiable everywhere else, and g attains a vertical tangent at b and is continuously differentiable everywhere else. Further, $a \neq b$. What can we say is **definitely true** about $f - g$? *Last year:* 5/15 *correct*
(A) $f - g$ has vertical tangents at a and b.
(B) $f - g$ has a vertical tangent at a and a vertical cusp at b.
(C) $f - g$ has a vertical cusp at a and a vertical tangent at b.
(D) $f - g$ has no vertical tangents and no vertical cusps.
(E) $f - g$ has either a vertical tangent or a vertical cusp at the points a and b, but it is not possible to be more specific without further information.

Your answer: ____________________________

(7) (**) Suppose f and g are continuous functions on $\mathbb{R}$, such that f is continuously differentiable everywhere and g is continuously differentiable everywhere except at c, where it has a vertical tangent. What can we say is **definitely true** about $f \circ g$? *Last year:* 3/15 *correct*
(A) It has a vertical tangent at c.
(B) It has a vertical cusp at c.
(C) It has either a vertical tangent or a vertical cusp at c.
(D) It has neither a vertical tangent nor a vertical cusp at c.
(E) We cannot say anything for certain.

Your answer: ____________________________

WORDS AND PICTURES: OCTOBER 26: CALCULUS IN THE REAL WORLD

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____
This is just a fun quiz.

(1) "The Reaction Time game is a game used to measure people's reaction time. When people start playing the game, their reaction time is in the 250-300ms range. We find that repeatedly playing the Reaction Time game causes the average reaction time to drop (allowing for random fluctuation as well as day-to-day variation on account of the person's physiological state). However, it does not approach zero but rather converges toward an *asymptotic* positive value of 150ms."

(2) "Our website traffic shows significant seasonal variation, with seasonal peaks during the US academic year and seasonal troughs during the summer vacation. We also have weekly periodic variation, with intra-week highs Monday - Thursday and lows Friday - Sunday. Controlling for seasonal variation, daily traffic to our website is growing by 1000 pageviews per day every year."

(3) "The instructor was sued under discrimination law because the letter grades he assigned on the test were not a *non-decreasing function* of the students' raw score on the test."

(4) "Opportunity cost and the reality of trade-offs mean that the common welfare will not be maximized if all resources are exclusively used in the production of wheat, nor if none are. Rather, the optimal amount of resources that ought to be devoted to wheat production lies in the middle. It is the middles, not the extremes, that are best for the common welfare."

(5) "The most pernicious forms of laziness and complacency arise when you are at a *local maximum* and are therefore unwilling to take a temporary dip in order to strive toward the bigger *absolute maximum*."

(6) "Every day, the daily sales at my lemonade stand go up by one, so the cumulative sales so far are a *quadratic function* of the number of days I have kept my lemonade stand open."

(7) "My Facebook friend count continues to grow, but the rate at which it is growing is steadily decreasing as I have to spend more and more time on work and less and less time on meeting new people outside my immediate friend circle."

(8) "In small quantities, enzyme C acts as a catalyst for its own creation (heard of autocatalysis?). However, once the quantity of enzyme C exceeds a certain threshold value, it acts as an inhibitor for its own creation. The biochemical reaction that produces enzyme C thus begins slowly, speeds up, and then reaches an *inflection point* after which it starts slowing down and eventually tapers off to zero. (Think of the graphs for the quantity of C produced and the rate of C produced per unit time)."

CLASS QUIZ: OCTOBER 28: INTEGRATION BASICS

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____________________

(1) Consider the function(s) $[0,1] \to \mathbb{R}$. **Identify the functions** for which the integral (using upper sums and lower sums) is not defined. *Last year:* 15/15 *correct*

(A) $f_1(x) := \begin{cases} 0, & 0 \le x < 1/2 \\ 1, & 1/2 \le x \le 1 \end{cases}$.

(B) $f_2(x) := \begin{cases} 0, & x \ne 0 \text{ and } 1/x \text{ is an integer} \\ 1, & \text{otherwise} \end{cases}$.

(C) $f_3(x) := \begin{cases} 0, & x \text{ rational} \\ 1, & x \text{ irrational} \end{cases}$

(D) All of the above

(E) None of the above

Your answer: ____________________

(2) (**) Suppose $a < b$. Recall that a *regular partition* into n parts of $[a,b]$ is a partition $a = x_0 < x_1 < \dots < x_{n-1} < x_n = b$ where $x_i - x_{i-1} = (b-a)/n$ for all $1 \le i \le n$. A partition P_1 is said to be a *finer partition* than a partition P_2 if the set of points of P_1 contains the set of points of P_2. Which of the following is a **necessary and sufficient condition** for the regular partition into m parts to be a *finer partition* than the regular partition into n parts? (Note: We'll assume that any partition is finer than itself). *Last year:* 5/15 *correct*

(A) $m \le n$

(B) $n \le m$

(C) m divides n (i.e., n is a multiple of m)

(D) n divides m (i.e., m is a multiple of n)

(E) m is a power of n

Your answer: ____________________

PLEASE TURN OVER FOR THIRD AND FOURTH QUESTIONS

(3) (**) For a partition $P = x_0 < x_1 < x_2 < \cdots < x_n$ of $[a, b]$ (with $x_0 = a$, $x_n = b$) define the norm $\|P\|$ as the maximum of the values $x_i - x_{i-1}$. Which of the following **is always true** for any continuous function f on $[a, b]$? *Last year:* 4/15 *correct*

(A) If P_1 is a finer partition than P_2, then $\|P_2\| \leq \|P_1\|$ (Here, *finer* means that, as a set, $P_2 \subseteq P_1$, i.e., all the points of P_2 are also points of P_1).
(B) If $\|P_2\| \leq \|P_1\|$, then $L_f(P_2) \leq L_f(P_1)$ (where L_f is the lower sum).
(C) If $\|P_2\| \leq \|P_1\|$, then $U_f(P_2) \leq U_f(P_1)$ (where U_f is the upper sum).
(D) If $\|P_2\| \leq \|P_1\|$, then $L_f(P_2) \leq U_f(P_1)$.
(E) All of the above.

Your answer: ____________________________

(4) (**) Suppose F and G are continuously differentiable functions on all of $\mathbb{R}$ (i.e., both F' and G' are continuous). Which of the following is **not necessarily true**? *Last year:* 0/15 *correct*

(A) If $F'(x) = G'(x)$ for all integers x, then $F - G$ is a constant function when restricted to integers, i.e., it takes the same value at all integers.
(B) If $F'(x) = G'(x)$ for all numbers x that are not integers, then $F - G$ is a constant function when restricted to the set of numbers x that are not integers.
(C) If $F'(x) = G'(x)$ for all rational numbers x, then $F - G$ is a constant function when restricted to the set of rational numbers.
(D) If $F'(x) = G'(x)$ for all irrational numbers x, then $F - G$ is a constant function when restricted to the set of irrational numbers.
(E) None of the above, i.e., they are all necessarily true.

Your answer: ____________________________

CLASS QUIZ: NOVEMBER 2: INTEGRATION

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) Suppose f and g are both functions on $\mathbb{R}$ with the property that f'' and g'' are both everywhere the zero function. For which of the following functions is the second derivative *not necessarily* the zero function everywhere? *Last year:* 14/15 *correct*
(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
(B) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(C) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(D) All of the above, i.e., the second derivative need not be identically zero for any of these functions.
(E) None of the above, i.e., for all these functions, the second derivative is the zero function.

Your answer: ____________________________

(2) Suppose f and g are both functions on $\mathbb{R}$ with the property that f''' and g''' are both everywhere the zero function. For which of the following functions is the third derivative *necessarily* the zero function everywhere? *Last year:* 12/15 *correct*
(A) $f + g$, i.e., the function $x \mapsto f(x) + g(x)$
(B) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(C) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(D) All of the above, i.e., the third derivative is identically zero for all of these functions.
(E) None of the above, i.e., the third derivative is not guaranteed to be the zero function for any of these.

Your answer: ____________________________

(3) Suppose f is a function on an interval $[a, b]$, that is continuous except at finitely many interior points $c_1 < c_2 < \dots < c_n$ ($n \geq 1$), where it has jump discontinuities (hence, both the left-hand limit and the right-hand limit exist but are not equal). Define $F(x) := \int_a^x f(t)\,dt$. Which of the following **is true**? *Last year:* 8/15 *correct*
(A) F is continuously differentiable on (a, b) and the derivative equals f wherever f is continuous.
(B) F is differentiable on (a, b) but the derivative is not continuous, and $F' = f$ on the entire interval.
(C) F has one-sided derivatives on (a, b) and the left-hand derivative of F at any point equals the left-hand limit of f at that point, while the right-hand derivative of F at any point equals the right-hand limit of f at that point.

(D) F has one-sided derivatives on all points of (a, b) except at the points $c_1, c_2, \ldots, c_n$; it is continuous at all these points but does not have one-sided derivatives.
(E) F is continuous at all points of (a, b) except at the points $c_1, c_2, \ldots, c_n$.

Your answer: ____________________________

(4) (**) For a continuous function f on $\mathbb{R}$ and a real number a, define $F_{f,a}(x) = \int_a^x f(t)\,dt$. Which of the following is **true**? *Last year:* 5/15 *correct*
(A) For every continuous function f and every real number a, $F_{f,a}$ is an antiderivative for f, and every antiderivative of f can be obtained in this way by choosing a suitably.
(B) For every continuous function f and every real number a, $F_{f,a}$ is an antiderivative for f, but it is not necessary that every antiderivative of f can be obtained in this way by choosing a suitably. (i.e., there are continuous functions f where not every antiderivative can be obtained in this way).
(C) For every continuous function f, every antiderivative of f can be written as $F_{f,a}$ for some suitable a, but there may be some choices of f and a for which $F_{f,a}$ is not an antiderivative of f.
(D) There may be some choices for f and a for which $F_{f,a}$ is not an antiderivative for f, and there may be some choices of f for which there exist antiderivatives that cannot be written in the form $F_{f,a}$.
(E) None of the above.

Your answer: ____________________________

(5) (**) Suppose F is a differentiable function on an open interval (a, b) and F' is not a continuous function. Which of these discontinuities can F' have? *Last year:* 0/15 *correct*
(A) A removable discontinuity (the limit exists and is finite but is not equal to the value of the function)
(B) An infinite discontinuity (one or both the one-sided limits is infinite)
(C) A jump discontinuity (both one-sided limits exist and are finite, but not equal)
(D) All of the above
(E) None of the above

Your answer: ____________________________

CLASS QUIZ: NOVEMBER 4: INTEGRATION

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

(1) Which of the following is an **antiderivative** of $x \cos x$?
(A) $x \sin x + \cos x$
(B) $x \sin x - \cos x$
(C) $-x \sin x + \cos x$
(D) $-x \sin x - \cos x$
(E) None of the above

Your answer: ____________________

(2) (*) Suppose F and G are two functions defined on $\mathbb{R}$ and k is a natural number such that the k^{th} derivatives of F and G exist and are equal on all of $\mathbb{R}$. Then, $F - G$ must be a polynomial function. What is the **maximum possible degree** of $F - G$? (Note: Assume constant polynomials to have degree zero)
(A) $k - 2$
(B) $k - 1$
(C) k
(D) $k + 1$
(E) There is no bound in terms of k.

Your answer: ____________________

(3) (**) Suppose f is a continuous function on $\mathbb{R}$. Clearly, f has antiderivatives on $\mathbb{R}$. For all but one of the following conditions, it is possible to guarantee, without any further information about f, that there exists an antiderivative F satisfying that condition. **Identify the exceptional condition** (i.e., the condition that it may not always be possible to satisfy).
(A) $F(1) = F(0)$.
(B) $F(1) + F(0) = 0$.
(C) $F(1) + F(0) = 1$.
(D) $F(1) = 2F(0)$.
(E) $F(1)F(0) = 0$.

Your answer: ____________________

(4) (**) Suppose $F(x) = \int_0^x \sin^2(t^2)\,dt$ and $G(x) = \int_0^x \cos^2(t^2)\,dt$. Which of the following **is true**?
(A) $F + G$ is the zero function.
(B) $F + G$ is a constant function with nonzero value.
(C) $F(x) + G(x) = x$ for all x.
(D) $F(x) + G(x) = x^2$ for all x.
(E) $F(x^2) + G(x^2) = x$ for all x.

Your answer: ____________________________

(5) (**) Suppose F is a function defined on $\mathbb{R} \setminus \{0\}$ such that $F'(x) = -1/x^2$ for all $x \in \mathbb{R} \setminus \{0\}$. Which of the following pieces of information is/are **sufficient** to determine F completely?
(A) The value of F at any two positive numbers.
(B) The value of F at any two negative numbers.
(C) The value of F at a positive number and a negative number.
(D) Any of the above pieces of information is sufficient, i.e., we need to know the value of F at any two numbers.
(E) None of the above pieces of information is sufficient.

Your answer: ____________________________

CLASS QUIZ: NOVEMBER 11: WHOPPERS

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ______________________________

(1) Suppose $g : \mathbb{R} \to \mathbb{R}$ is a continuous function such that $\lim_{x \to 0} g(x)/x^2 = A$ for some constant $A \neq 0$. What is $\lim_{x \to 0} g(g(x))/x^4$?
   (A) A
   (B) A^2
   (C) A^3
   (D) $A^2 g(A)$
   (E) $g(A)/A^2$
   Your answer: ______________________

(2) Which of the following statements is **always true**? *Exact replica of a past question.*
   (A) The range of a continuous nonconstant function on a closed bounded interval (i.e., an interval of the form $[a, b]$) is a closed bounded interval (i.e., an interval of the form $[m, M]$).
   (B) The range of a continuous nonconstant function on an open bounded interval (i.e., an interval of the form (a, b)) is an open bounded interval (i.e., an interval of the form (m, M)).
   (C) The range of a continuous nonconstant function on a closed interval that may be bounded or unbounded (i.e., an interval of the form $[a, b]$, $[a, \infty)$, $(-\infty, a]$, or $(-\infty, \infty)$) is also a closed interval that may be bounded or unbounded.
   (D) The range of a continuous nonconstant function on an open interval that may be bounded or unbounded (i.e., an interval of the form (a, b),(a, ∞), $(-\infty, a)$, or $(-\infty, \infty)$), is also an open interval that may be bounded or unbounded.
   (E) None of the above.
   Your answer: ______________________

(3) Suppose f is a continuously differentiable function on $\mathbb{R}$ and $c \in \mathbb{R}$. Which of the following implications is **false**? *Similar to a past question.*
   (A) If f has mirror symmetry about $x = c$, f' has half turn symmetry about $(c, f'(c))$.
   (B) If f has half turn symmetry about $(c, f(c))$, f' has mirror symmetry about $x = c$.
   (C) If f' has mirror symmetry about $x = c$, f has half turn symmetry about $(c, f(c))$.
   (D) If f' has half turn symmetry about $(c, f'(c))$, f has mirror symmetry about $x = c$.
   (E) None of the above, i.e., they are all true.
   Your answer: ______________________

(4) Consider the function $f(x) := \{ \begin{array}{rl} x, & 0 \le x \le 1/2 \\ x - (1/5), & 1/2 < x \le 1 \end{array}$. Define by $f^{[n]}$ the function obtained by iterating f n times, i.e., the function $f \circ f \circ f \circ \dots \circ f$ where f occurs n times. What is the smallest n for which $f^{[n]} = f^{[n+1]}$? *Similar to a question on the previous midterm.*
   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5
   Your answer: ______________________

(5) With f as in the previous question, what is the set of points in $(0, 1)$ where $f \circ f$ is not continuous?
   (A) 0.5 only
   (B) 0.5 and 0.7

(C) 0.5, 0.7, and 0.9
(D) 0.7 and 0.9
(E) 0.9 only

Your answer: ____________________________

(6) Consider the graph of the function $f(x) := x\sin(1/(x^2-1))$. What can we say about the vertical and horizontal asymptotes?
(A) The graph has vertical asymptotes at $x = +1$ and $x = -1$ and horizontal asymptote (in both directions) $y = 0$.
(B) The graph has vertical asymptotes at $x = +1$ and $x = -1$ and horizontal asymptote (in both directions) $y = 1$.
(C) The graph has no vertical asymptotes and horizontal asymptote (in both directions) $y = 0$.
(D) The graph has no vertical asymptotes and horizontal asymptote (in both directions) $y = 1$.
(E) The graph has no vertical or horizontal asymptotes.

Your answer: ____________________________

(7) Suppose f and g are increasing functions from $\mathbb{R}$ to $\mathbb{R}$. Which of the following functions is *not* guaranteed to be an increasing functions from $\mathbb{R}$ to $\mathbb{R}$? *An exact replica of a past question.*
(A) $f+g$
(B) $f \cdot g$
(C) $f \circ g$
(D) All of the above, i.e., none of them is guaranteed to be increasing.
(E) None of the above, i.e., they are all guaranteed to be increasing.

Your answer: ____________________________

(8) Suppose F and G are continuously differentiable functions on all of $\mathbb{R}$ (i.e., both F' and G' are continuous). Which of the following is **not necessarily true**? *Exact replica of a previous question.*
(A) If $F'(x) = G'(x)$ for all integers x, then $F-G$ is a constant function when restricted to integers, i.e., it takes the same value at all integers.
(B) If $F'(x) = G'(x)$ for all numbers x that are not integers, then $F-G$ is a constant function when restricted to the set of numbers x that are not integers.
(C) If $F'(x) = G'(x)$ for all rational numbers x, then $F-G$ is a constant function when restricted to the set of rational numbers.
(D) If $F'(x) = G'(x)$ for all irrational numbers x, then $F-G$ is a constant function when restricted to the set of irrational numbers.
(E) None of the above, i.e., they are all necessarily true.

Your answer: ____________________________

(9) Consider the four functions $\sin(\sin x)$, $\sin(\cos x)$, $\cos(\sin x)$, and $\cos(\cos x)$. Which of the following statements are true about their periodicity?
(A) All four functions are periodic with a period of 2π.
(B) All four functions are periodic with a period of π.
(C) $\sin(\sin x)$ and $\sin(\cos x)$ have a period of π, whereas $\cos(\sin x)$ and $\cos(\cos x)$ have a period of 2π.
(D) $\cos(\sin x)$ and $\cos(\cos x)$ have a period of π, whereas $\sin(\sin x)$ and $\sin(\cos x)$ have a period of 2π.
(E) $\sin(\sin x)$ has a period of 2π, the other three functions have a period of π.

Your answer: ____________________________

CLASS QUIZ: NOVEMBER 16: VOLUME

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) Oblique cylinder:Right cylinder:: *Last year:* 14/16 *correct*
- (A) Rectangle:Square
- (B) Parallelogram:Rectangle
- (C) Disk:Circle
- (D) Triangle:Rectangle
- (E) Triangle:Square

Your answer: ____________________________

(2) Right circular cone:Right circular cylinder:: *Last year:* 13/16 *correct*
- (A) Triangle:Square
- (B) Rectangle:Square
- (C) Isosceles triangle:Equilateral triangle
- (D) Isosceles triangle:Rectangle
- (E) Isosceles triangle:Square

Your answer: ____________________________

(3) Circular disk:Circle:: *Last year:* 8/16 *correct*
- (A) Hollow cylinder:Solid cylinder
- (B) Solid cylinder:Hollow cylinder
- (C) Cube:Cuboid (cuboid is a term for rectangular prism)
- (D) Cube:Square
- (E) Cube:Sphere

Your answer: ____________________________

(4) Circular disk:Line segment:: *Last year:* 14/16 *correct*
- (A) Solid sphere:Circular disk
- (B) Circle:Rectangle
- (C) Sphere:Cube
- (D) Cube:Right circular cylinder
- (E) Square:Triangle

Your answer: ____________________________

(5) Suppose a filled triangle ABC in the plane is revolved about the side AB. Which of the following best describes the solid of revolution thus obtained if both the angles A and B are acute (ignoring issues of boundary inclusion/exclusion)? *Last year:* 13/16 *correct*

(A) It is a right circular cone.
(B) It is the union of two right circular cones sharing a common disk as base.
(C) It is the set difference of two right circular cones sharing a common disk as base.
(D) It is the union of two right circular cones sharing a common vertex.
(E) It is the set difference of two right circular cones sharing a common vertex.

Your answer: ____________________________

(6) Suppose a filled triangle ABC in the plane is revolved about the side AB. Which of the following best describes the solid of revolution thus obtained if the angle A is obtuse (ignoring issues of boundary inclusion/exclusion)? *Last year:* 9/16 *correct*

(A) It is a right circular cone.
(B) It is the union of two right circular cones sharing a common disk as base.
(C) It is the set difference of two right circular cones sharing a common disk as base.
(D) It is the union of two right circular cones sharing a common vertex.
(E) It is the set difference of two right circular cones sharing a common vertex.

Your answer: ____________________________

(7) What is the volume of the solid of revolution obtained by revolving the filled triangle ABC about the side AB, if the length of the base AB is b and the height corresponding to this base is h? *Last year:* 10/16 *correct*

(A) $(1/6)\pi b^{3/2}h^{3/2}$
(B) $(1/3)\pi b^2 h$
(C) $(1/3)\pi bh^2$
(D) $(2/3)\pi b^2 h$
(E) $(2/3)\pi bh^2$

Your answer: ____________________________

For the next two questions, suppose Ω is a region in a plane Π and ℓ is a line on Π such that Ω lies completely on one side of ℓ (in particular, it does not intersect ℓ). Let Γ be the solid of revolution obtained by revolving Ω about ℓ. Suppose further that the intersection of Ω with any line perpendicular to ℓ is either empty or a point or a line segment.

(8) (*) What is the intersection of Γ with Π (your answer should be always true)? *Last year:* 6/16 *correct*

(A) It is precisely Ω.
(B) It is the union of Ω and a translate of Ω along a direction perpendicular to ℓ.
(C) It is the union of Ω and the reflection of Ω about ℓ.
(D) It is either empty or a rectangle whose dimensions depend on Ω.
(E) It is either empty or a circle or an annulus whose inner and outer radius depend on Ω.

Your answer: ____________________________

(9) What is the intersection of Γ with a plane perpendicular to ℓ (your answer should be always true)? *Last year:* 9/16 *correct*

(A) It is precisely Ω.
(B) It is the union of Ω and a translate of Ω along a direction perpendicular to ℓ.
(C) It is the union of Ω and the reflection of Ω about ℓ.
(D) It is either empty or a rectangle whose dimensions depend on Ω.
(E) It is either empty or a circle or an annulus whose inner and outer radius depend on Ω.

Your answer: ____________________________

(10) (*) Consider a fixed equilateral triangle ABC. Now consider, for any point D outside the plane of ABC, the solid tetrahedron $ABCD$. This is the solid bounded by the triangles ABC, BCD, ACD, and ABD. The volume of this solid depends on D. What specific information about D completely determines the volume? *Last year:* 7/16 *correct*

(A) The perpendicular distance from D to the plane of the triangle ABC.
(B) The minimum of the distances from D to points in the filled triangle ABC.
(C) The location of the point E in the plane of triangle ABC that is the foot of the perpendicular from D to ABC.
(D) The distance from D to the center of ABC (here, you can take the center as any of the notions of center since ABC is equilateral).
(E) None of the above.

Your answer: ____________________________

(11) (**) For $r > 0$, consider the region $\Omega_r(a)$ bounded by the x-axis, the curve $y = x^{-r}$, and the lines $x = 1$ and $x = a$ with $a > 1$. Let $V_r(a)$ be the volume of the region obtained by revolving $\Omega_r(a)$ about the x-axis. What is the precise set of values of r for which $\lim_{a\to\infty} V_r(a)$ is finite? *Last year:* 3/16 *correct*

(A) All $r > 0$
(B) $r > 1/2$
(C) $r > 1$
(D) $r > 2$
(E) No value of r

Your answer: ____________________________

CLASS QUIZ: NOVEMBER 21: ONE-ONE FUNCTIONS

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) For one of these function types for a continuous function from $\mathbb{R}$ to $\mathbb{R}$, it is *possible* to also be a one-to-one function. What is that function type? *Last year:* 15/15 *correct*
   (A) Function whose graph has mirror symmetry about a vertical line.
   (B) Function whose graph has half turn symmetry about a point on it.
   (C) Periodic function.
   (D) Function having a point of local minimum.
   (E) Function having a point of local maximum.

   Your answer: ____________________________

(2) (**) Suppose f, g, and h are continuous one-to-one functions whose domain and range are both $\mathbb{R}$. **What can we say** about the functions $f+g$, $f+h$, and $g+h$? *Last year:* 2/15 *correct*
   (A) They are all continuous one-to-one functions with domain $\mathbb{R}$ and range $\mathbb{R}$.
   (B) At least two of them are continuous one-to-one functions with domain $\mathbb{R}$ and range $\mathbb{R}$ – however, we cannot say more.
   (C) At least one of them is a continuous one-to-one function with domain $\mathbb{R}$ and range $\mathbb{R}$ – however, we cannot say more.
   (D) Either all three sums are continuous one-to-one functions whose domain and range are both $\mathbb{R}$, or none is.
   (E) It is possible that none of the sums is a continuous one-to-one function whose domain and range are both $\mathbb{R}$; it is also possible that one, two, or all the sums are continuous one-to-one functions whose domain and range are both $\mathbb{R}$.

   Your answer: ____________________________

(3) (**) Suppose f is a one-to-one function with domain a closed interval $[a,b]$ and range a closed interval $[c,d]$. Suppose t is a point in (a,b) such that f has left hand derivative l and right-hand derivative r at t, with both l and r nonzero. What is the left hand derivative and right hand derivative to f^{-1} at $f(t)$? *Last year:* 6/15 *correct*
   (A) The left hand derivative is $1/l$ and the right hand derivative is $1/r$.
   (B) The left hand derivative is $-1/l$ and the right hand derivative is $-1/r$.
   (C) The left hand derivative is $1/r$ and the right hand derivative is $1/l$.
   (D) The left hand derivative is $-1/r$ and the right hand derivative is $-1/l$.
   (E) The left hand derivative is $1/l$ and the right hand derivative is $1/r$ if $l > 0$, otherwise the left hand derivative is $1/r$ and the right hand derivative is $1/l$.

   Your answer: ____________________________

(4) (**) Which of these functions is one-to-one? *Last year:* 2/15 *correct*

(A) $f_1(x) := \{ \begin{array}{rl} x, & x \text{ rational} \\ x^2, & x \text{ irrational} \end{array}$

(B) $f_2(x) := \{ \begin{array}{rl} x, & x \text{ rational} \\ x^3, & x \text{ irrational} \end{array}$

(C) $f_3(x) := \{ \begin{array}{rl} x, & x \text{ rational} \\ 1/(x-1), & x \text{ irrational} \end{array}$

(D) All of the above

(E) None of the above

Your answer: ______________________________

(5) (**) Consider the following function $f : [0,1] \to [0,1]$ given by $f(x) := \{ \begin{array}{rl} \sin(\pi x/2), & 0 \leq x \leq 1/2 \\ \sqrt{x}, & 1/2 < x \leq 1 \end{array}$.

What is the correct expression for $(f^{-1})'(1/2)$? *Last year:* 1/15 *correct*

(A) It does not exist, since the two-sided derivatives of f at $1/2$ do not match.

(B) $\sqrt{2}$

(C) $2\sqrt{2}/\pi$

(D) $4/\pi$

(E) $4/(\sqrt{3}\pi)$

Your answer: ______________________________

CLASS QUIZ: NOVEMBER 23: MEMORY LANE

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

(1) For which of the following specifications is there **no continuous function** satisfying the specifications?
(A) Domain $[0,1]$ and range $[0,1]$
(B) Domain $[0,1]$ and range $(0,1)$
(C) Domain $(0,1)$ and range $[0,1]$
(D) Domain $(0,1)$ and range $(0,1)$
(E) None of the above, i.e., we can get a continuous function for each of the specifications.

Your answer: ____________________________

(2) Suppose f and g are continuous functions on $\mathbb{R}$, such that f is continuously differentiable everywhere and g is continuously differentiable everywhere except at c, where it has a vertical tangent. What can we say is **definitely true** about $f \circ g$?
(A) It has a vertical tangent at c.
(B) It has a vertical cusp at c.
(C) It has either a vertical tangent or a vertical cusp at c.
(D) It has neither a vertical tangent nor a vertical cusp at c.
(E) We cannot say anything for certain.

Your answer: ____________________________

(3) Consider the function $p(x) := x^{2/3}(x-1)^{3/5} + (x-2)^{7/3}(x-5)^{4/3}(x-6)^{4/5}$. For what values of x does the graph of p have a vertical cusp at $(x, p(x))$?
(A) $x = 0$ only.
(B) $x = 0$ and $x = 5$ only.
(C) $x = 5$ and $x = 6$ only.
(D) $x = 0$ and $x = 6$ only.
(E) $x = 0$, $x = 5$, and $x = 6$.

Your answer: ____________________________

(4) Consider the function $f(x) := \{ \begin{array}{ll} x, & 0 \le x \le 1/2 \\ x^2, & 1/2 < x \le 1 \end{array}$. What is $f \circ f$?
(A) $x \mapsto \{ \begin{array}{ll} x, & 0 \le x \le 1/2 \\ x^4, & 1/2 < x \le 1 \end{array}$
(B) $x \mapsto \{ \begin{array}{ll} x, & 0 \le x \le 1/2 \\ x^2, & 1/2 < x \le 1 \end{array}$

(C) $x \mapsto \{ \begin{array}{ll} x, & 0 \leq x \leq 1/2 \\ x^2, & 1/2 < x \leq 1/\sqrt{2} \\ x^4, & 1/\sqrt{2} < x \leq 1 \end{array}$

(D) $x \mapsto \{ \begin{array}{ll} x, & 0 \leq x \leq 1/\sqrt{2} \\ x^2, & 1/\sqrt{2} < x \leq 1 \end{array}$

(E) $x \mapsto \{ \begin{array}{ll} x, & 0 \leq x \leq 1/\sqrt{2} \\ x^4, & 1/\sqrt{2} < x \leq 1 \end{array}$

Your answer: ____________________________

(5) Suppose f and g are functions $(0,1)$ to $(0,1)$ that are both right continuous on $(0,1)$. Which of the following is *not* guaranteed to be right continuous on $(0,1)$?
(A) $f+g$, i.e., the function $x \mapsto f(x)+g(x)$
(B) $f-g$, i.e., the function $x \mapsto f(x)-g(x)$
(C) $f \cdot g$, i.e., the function $x \mapsto f(x)g(x)$
(D) $f \circ g$, i.e., the function $x \mapsto f(g(x))$
(E) None of the above, i.e., they are all guaranteed to be right continuous functions

Your answer: ____________________________

(6) For a partition $P = x_0 < x_1 < x_2 < \cdots < x_n$ of $[a,b]$ (with $x_0 = a$, $x_n = b$) define the norm $\|P\|$ as the maximum of the values $x_i - x_{i-1}$. Which of the following **is always true** for any continuous function f on $[a,b]$? (5 points)
(A) If P_1 is a finer partition than P_2, then $\|P_2\| \leq \|P_1\|$ (Here, *finer* means that, as a set, $P_2 \subseteq P_1$, i.e., all the points of P_2 are also points of P_1).
(B) If $\|P_2\| \leq \|P_1\|$, then $L_f(P_2) \leq L_f(P_1)$ (where L_f is the lower sum).
(C) If $\|P_2\| \leq \|P_1\|$, then $U_f(P_2) \leq U_f(P_1)$ (where U_f is the upper sum).
(D) If $\|P_2\| \leq \|P_1\|$, then $L_f(P_2) \leq U_f(P_1)$.
(E) All of the above.

Your answer: ____________________________

(7) A disk of radius r in the xy-plane is translated parallel to itself with its center moving in the yz-plane along the semicircle $y^2 + z^2 = R^2, y \geq 0$. The solid thus obtained can be thought of as a *cylinder of bent spine* with cross sections being disks of radius r along the xy-plane and the centers forming a semicircle of radius R in the yz-plane, with the z-value ranging from $-R$ to R. What is the volume of this solid?
(A) $2\pi r^2 R$
(B) $\pi^2 r^2 R$
(C) $2\pi r R^2$
(D) $\pi^2 r R^2$
(E) $\pi^2 R^3$

Your answer: ____________________________

CLASS QUIZ: NOVEMBER 28: LOGARITHM AND EXPONENTIAL

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

Note: I didn't administer this quiz last year, so I don't have data on the level of difficulty of the questions. Thus, the starring of questions is based on guesswork.

(1) Consider the function $f(x) := \exp(5 \ln x)$ defined for $x \in (0, \infty)$. How does $f(x)$ grow as a function of x?
 (A) As a linear function
 (B) As a polynomial function but faster than a linear function
 (C) Faster than a polynomial function but slower than an exponential function
 (D) As an exponential function, i.e., $x \mapsto \exp(kx)$ for some $k > 0$
 (E) Faster than an exponential function

 Your answer: ____________________________

(2) Consider the function $f(x) := \ln(5 \exp x)$ for $x \in (0, \infty)$. How does $f(x)$ grow as a function of x?
 (A) As a linear function
 (B) As a polynomial function but faster than a linear function
 (C) Faster than a polynomial function but slower than an exponential function
 (D) As an exponential function, i.e., $x \mapsto \exp(kx)$ for some $k > 0$
 (E) Faster than an exponential function

 Your answer: ____________________________

(3) Consider the function $f(x) := \ln((\exp x)^5)$ defined for $x \in (0, \infty)$. How does $f(x)$ grow as a function of x?
 (A) As a linear function
 (B) As a polynomial function but faster than a linear function
 (C) Faster than a polynomial function but slower than an exponential function
 (D) As an exponential function, i.e., $x \mapsto \exp(kx)$ for some $k > 0$
 (E) Faster than an exponential function

 Your answer: ____________________________

PLEASE TURN OVER FOR REMAINING QUESTIONS

(4) (*) Consider the function $f(x) := \exp((\ln x)^5)$ defined for $x \in (0, \infty)$. How does $f(x)$ grow as a function of x?
(A) As a linear function
(B) As a polynomial function but faster than a linear function
(C) Faster than a polynomial function but slower than an exponential function
(D) As an exponential function, i.e., $x \mapsto \exp(kx)$ for some $k > 0$
(E) Faster than an exponential function

Your answer: ____________________________

(5) (*) *Consumption smoothing*: A certain measure of happiness is found to be a logarithmic function of consumption, i.e., the happiness level H of a person is found to be of the form $H = a + b \ln C$ where C is the person's current consumption level, and a and b are positive constants independent of the consumption level.

The person has a certain total consumption C_{tot} to be split within two years, year 1 and year 2, i.e., $C_{tot} = C_1 + C_2$. Thus, the person's happiness level in year 1 is $H_1 = a + b \ln C_1$ and the person's happiness level in year 2 is $H_2 = a + b \ln C_2$. How would the person choose to split consumption between the two years to maximize average happiness across the years?
(A) All the consumption in either one year
(B) Equal amount of consumption in the two years
(C) Consume twice as much in one year as in the other year
(D) Consumption in the two years is in the ratio $a : b$
(E) It does not matter because any choice of split of consumption level between the two years produces the same average happiness

Your answer: ____________________________

(6) (*) *Income inequality and subjective well being*: Subjective well being *across* individuals is found to be logarithmically related to income. Every doubling of income is found to increase an individuals' measured subjective well being by 0.3 points on a certain scale. *Holding total income across two individuals constant*, how should that income be divided between the two individuals to maximize their average subjective well being?
(A) All the income goes to one person
(B) Both earn the exact same income
(C) One person earns twice as much as the other
(D) One person earns 0.3 times as much as the other
(E) It does not matter because the average subjective well being is independent of the distribution of income.

Your answer: ____________________________

TAKE HOME CLASS QUIZ: TURN IN NOVEMBER 30: INTEGRATION

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____

THIS IS A TAKE HOME QUIZ.

FEEL FREE TO DISCUSS ALL QUESTIONS, BUT ATTEMPT THEM YOURSELF FIRST.

(1) What is the limit $\lim_{x\to\infty}\left[\left(\int_0^x \sin^2\theta d\theta\right)/x\right]$? *Last year:* 13/16 *correct*
   (A) $1/2$
   (B) 1
   (C) $1/\pi$
   (D) $2/\pi$
   (E) $1/(2\pi)$

   Your answer: ____________________________

(2) Consider the substitution $u = -1/x$ for the integral $\int \frac{dx}{x^2+1}$. What is the **new integral**? *Last year:* 8/16 *correct*
   (A) $\int \frac{du}{u(u^2+1)}$
   (B) $\int \frac{du}{u^2+1}$
   (C) $\int \frac{udu}{u^2+1}$
   (D) $\int \frac{u^2du}{u^2+1}$
   (E) $\int \frac{u^2du}{(u^2+1)^2}$

   Your answer: ____________________________

(3) *Hard*: What is the **value** of $c \in (0,\infty)$ such that $\int_0^c \frac{dx}{x^2+1} = \lim_{a\to\infty}\int_c^a \frac{dx}{x^2+1}$? *Last year:* 8/16 *correct*
   (A) $\frac{1}{\sqrt{3}}$
   (B) $\frac{1}{\sqrt{2}}$
   (C) 1
   (D) $\sqrt{2}$
   (E) $\sqrt{3}$

   Your answer: ____________________________

(4) Suppose f is a continuous nonconstant even function on $\mathbb{R}$. Which of the following is **true**? *Last year:* 4/16 *correct*
   (A) Every antiderivative of f is an even function.
   (B) f has exactly one antiderivative that is an even function.
   (C) Every antiderivative of f is an odd function.

(D) f has exactly one antiderivative that is an odd function.
(E) None of the antiderivatives of f is either an even or an odd function.

Your answer: ____________________________

(5) Suppose f is a continuous nonconstant odd function on $\mathbb{R}$. Which of the following is **true**? *Last year:* 13/16 *correct*
(A) Every antiderivative of f is an even function.
(B) f has exactly one antiderivative that is an even function.
(C) Every antiderivative of f is an odd function.
(D) f has exactly one antiderivative that is an odd function.
(E) None of the antiderivatives of f is either an even or an odd function.

Your answer: ____________________________

(6) Suppose f is a continuous nonconstant periodic function on $\mathbb{R}$ with period h. Which of the following is **true**? *Last year:* 5/16 *correct*
(A) Every antiderivative of f is a periodic function with period h, regardless of the choice of f.
(B) For some choices of f, every antiderivative of f is a periodic function; for all others, f has no periodic antiderivative.
(C) f has exactly one periodic antiderivative for every choice of f.
(D) For some choices of f, f has exactly one periodic antiderivative; for all others, f has no periodic antiderivative.
(E) Regardless of the choice of f, no antiderivative of f can be periodic.

Your answer: ____________________________

(7) Consider a continuous increasing function f defined on the nonnegative real numbers. Define $m_f(a)$, for $a > 0$, as the unique value $c \in [0, a]$ such that $f(c)$ is the mean value of f on the interval $[0, a]$.

If $f(x) := x^n$, n an integer greater than 1, what kind of function is m_f (your answer should be valid for all n)? *Last year:* 1/16 *correct*
(A) $m_f(a)$ is a constant λ dependent on n but independent of a.
(B) It is a function of the form $m_f(a) = \lambda a$, where λ is a constant depending on n.
(C) It is a function of the form $m_f(a) = \lambda a^{n-1}$, where λ is a constant depending on n.
(D) It is a function of the form $m_f(a) = \lambda a^n$, where λ is a constant depending on n.
(E) It is a function of the form $m_f(a) = \lambda a^{n+1}$, where λ is a constant depending on n.

Your answer: ____________________________