CLASS QUIZ: OCTOBER 21: MAX-MIN PROBLEMS

MATH 152, SECTION 55 (VIPUL NAIK)

Your name (print clearly in capital letters): ____
Note: Questions 5-7 have only four options.

(1) Consider all the rectangles with perimeter equal to a fixed length $p > 0$. Which of the following **is true** for the unique rectangle which is a square, compared to the other rectangles? *Last year:* 15/15 *correct*
(A) It has the largest area and the largest length of diagonal.
(B) It has the largest area and the smallest length of diagonal.
(C) It has the smallest area and the largest length of diagonal.
(D) It has the smallest area and the smallest length of diagonal.
(E) None of the above.

Your answer: ____________________________

(2) Suppose the total perimeter of a square and an equilateral triangle is L. (We can choose to allocate all of L to the square, in which case the equilateral triangle has side zero, and we can choose to allocate all of L to the equilateral triangle, in which case the square has side zero). Which of the following statements **is true** for the sum of the areas of the square and the equilateral triangle? (The area of an equilateral triangle is $\sqrt{3}/4$ times the square of the length of its side). *Last year:* 9/15 *correct*
(A) The sum is minimum when all of L is allocated to the square.
(B) The sum is maximum when all of L is allocated to the square.
(C) The sum is minimum when all of L is allocated to the equilateral triangle.
(D) The sum is maximum when all of L is allocated to the equilateral triangle.
(E) None of the above.

Your answer: ____________________________

(3) Suppose x and y are positive numbers such as $x + y = 12$. For **what values** of x and y is x^2y maximum? *Last year:* 12/15 *correct*
(A) $x = 3$, $y = 9$
(B) $x = 4$, $y = 8$
(C) $x = 6$, $y = 6$
(D) $x = 8$, $y = 4$
(E) $x = 9$, $y = 3$

Your answer: ____________________________

(4) (**) Consider the function $p(x) := x^2+bx+c$, with x restricted to *integer inputs*. Suppose b and c are integers. The minimum value of p is attained either at a single integer or at two consecutive integers. Which of the following is a **sufficient condition** for the minimum to occur at two consecutive integers? *Last year:* 4/15 *correct*

(A) b is odd
(B) b is even
(C) c is odd
(D) c is even
(E) None of these conditions is sufficient.

Your answer: ____________________________

(5) (**) Consider a hollow cylinder with no top and bottom and total curved surface area S. What can we say about the **maximum and minimum** possible values of the **volume**? (for radius r and height h, the curved surface area is $2\pi rh$ and the volume is $\pi r^2 h$). *Last year:* 6/15 *correct*

(A) The volume can be made arbitrarily small (i.e., as close to zero as we desire) and arbitrarily large (i.e., as large as we want).
(B) There is a positive minimum value for the volume, but it can be made arbitrarily large.
(C) There is a finite maximum value for the volume, but it can be made arbitrarily small.
(D) There is both a finite positive minimum and a finite positive maximum for the volume.

Your answer: ____________________________

(6) Consider a hollow cylinder with a bottom but no top and total surface area (curved surface plus bottom) S. What can we say about the **maximum and minimum** possible values of the **volume**? (for radius r and height h, the curved surface area is $2\pi rh$ and the volume is $\pi r^2 h$). *Last year:* 8/15 *correct*

(A) The volume can be made arbitrarily small (i.e., as close to zero as we desire) and arbitrarily large (i.e., as large as we want).
(B) There is a positive minimum value for the volume, but it can be made arbitrarily large.
(C) There is a finite maximum value for the volume, but it can be made arbitrarily small.
(D) There is both a finite positive minimum and a finite positive maximum for the volume.

Your answer: ____________________________

(7) (**) Consider a hollow cylinder with a bottom and a top and total surface area (curved surface plus bottom and top) S. What can we say about the **maximum and minimum** possible values of the **volume**? (for radius r and height h, the curved surface area is $2\pi rh$ and the volume is $\pi r^2 h$). *Last year:* 5/15 *correct*

(A) The volume can be made arbitrarily small (i.e., as close to zero as we desire) and arbitrarily large (i.e., as large as we want).
(B) There is a positive minimum value for the volume, but it can be made arbitrarily large.
(C) There is a finite maximum value for the volume, but it can be made arbitrarily small.
(D) There is both a finite positive minimum and a finite positive maximum for the volume.

Your answer: ____________________________